an extract from:

Pitch, Pi, and Other Musical Paradoxes (A Practical Guide to Natural Microtonality)

by Charles E. H. Lucy ISBN 0-9512879-0-7

Chapter One. (First published in *Music Teacher* magazine, January 1988, London)

IS THIS THE LOST MUSIC OF THE SPHERES?

After twenty five years of playing, I realised that I was still unable to tune any guitar so that it sounded in tune for both an open G major and an open E major. I tried tuning forks, pitch pipes, electronic tuners and harmonics at the seventh, fifth and twelfth frets. I sampled the best guitars I could find, but none would "sing" for both chords. At the risk of appearing tone deaf, I confessed my incompetence to a few trusted musical friends, secretly hoping that someone would initiate me into the secret of tuning guitars, so that they would "sing" for all chords. A few admitted that they had the same problem, but none would reveal the secret.

I had read how the position of the frets was calculated from the twelfth root of two, so that each of the twelve semitones in an octave had equal intervals of 100 cents. It is the equality of these intervals, which allows us to easily modulate or transpose into any of the twelve keys.

Without realising it, I had begun a quest for the lost music of the spheres. I still secretly doubted my musical ear, but rationalised my search by professing an interest in microtonal music, and claimed to be searching for the next step in the evolution of music. First stop library, where I found Helmholtz's book *On The Sensations Of Tone*. He introduced me to thousands of alternative tunings. The only universal 'truth', on which all tunings agreed was that halving the length of a string doubles the frequency, and produces a ratio of 2.00000:1 which is known as an octave. This octave was subdivided in three basic ways;

1) By a geometric progression, with any number of equal intervals.Eg. 12 (as on conventional guitars) [100 cents per semitone or interval]; 31 as advocated by Huyghens (1629-1695) [38.71 cents per interval], or 53 by Mercator and Bosanquet (1876 Treatise) [22.64 cents];

2) By low whole number ratios. Eg. (Just Intonation) 3:2 for the Vth; 5:4 for the major third etc.

3) By cumulative fifths. Eg. Pythagorean Tuning 3:2 for the Vth; but 81:64 for the major third.

There are also hybrids of the other three, Eg. Meantone Temperaments .

If the only point of agreement is that the octave ratio should be exactly two; how do we explain the phenomenon of stretched octaves, used by some piano tuners?

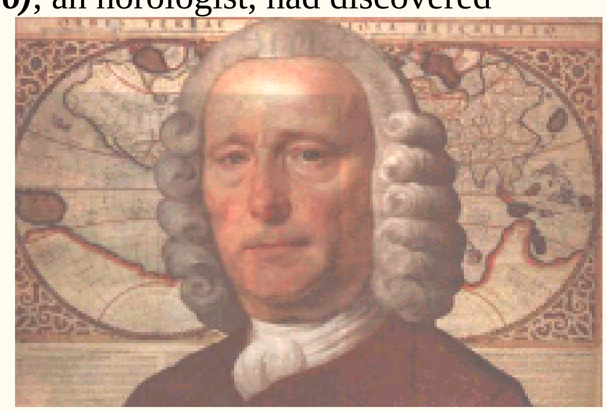
The Pythagorean system , instead of arriving at the octave after 12 steps i.e. (3^12)/(2^18) becomes 531441/262144 = 2.0272865 instead of 2.0000 as we assume when tuning a guitar by fifths at the seventh fret. This difference or error is known as *Pythagoras' lemma*. Every system seems to be an imperfect compromise, which is probably why mathematicians and musicians have devoted millions of hours to searching for the perfect scale.

Initially, the idea of 53 notes on a geometric progression seemed to be a sensible solution, and I read that Bosanquet's harmonium on this scale had been in the Kensington Science Museum since the 1880's. I went to find it, but it was in storage. Instead I found Mr. Chew. I told him of my quest and that I had a hunch that the solution was in some way connected with the music of the spheres and the Greek letter " Pi ".

π

"That's what Harrison thought."

I enquired further and discovered that **John Harrison *(1693-1776)***, an horologist, had discovered longitude and won a £20,000 prize from Parliament after the personal intervention of George III. I was directed to the Clockmakers' Library in the Guildhall, and there found a treasured copy of *A Description concerning such Mechanism as will afford a nice, or true Mensuration of Time; together with Some Account of the Attempts for the Discovery of the Longitude by the Moon; and also An Account of the Discovery of the Scale of Musick*, harrison.zip only to be refused permission to photocopy any of it, due to its antiquity. I eventually acquired photocopies of the relevant pages from another source. Harrison had written in such an obscure style that



I suspected that he intended to hide its secrets from all but the most diligent enquirer.

The essence of what Harrison said is as follows:

"The *natural scale of music* is associated with the ratio of the diameter of a circle to its circumference." (i.e. pi = 3.14159265358979323846 etc.)

"This scale is based on two intervals;"

1) The *Larger note* as he calls it; This is a ratio of 2 to the 2*pi root of 2, or in BASIC computer terms 2^(1/(2*pi)), which equals a ratio of 1.116633 or 190.9858 cents, approximately 1.91 frets on a conventional guitar. (L)

2) The *lesser note*, which is half the difference between five *Larger notes* (5L) and an octave. i.e.(2/(2^(1/(2*pi)))^5)^(1/2), giving a ratio of 1.073344 or 122.5354 cents, an interval of approx. 1.23 frets. (s)

The equivalent of the fifth (i.e. seventh fret on guitar) is composed of three Large (3L) plus one small note (s) i.e. (3L+s) = (190.986*3) + (122.535) = 695.493 cents or ratio of 1.494412.

The equivalent of the fourth (IV) (fifth fret) is 2L+s = 504.507 cents.

To convert between ratio and cents
Enter a number in either field, then click outside the text box.

Ratio: 1.5
Cents: 701.955

Harrison discovered this scale experimenting with monochords and a viol, and trained a church choir in Lincolnshire to sing it.

From this information I constructed a number of computer programs to explore all the possible permutations and combinations which could generate a practical scale. My problem was to decide which of the hundreds of intervals which I calculated by addition and subtraction of multiples of the Large (L) and small (s) interval should be used. Faced with pages of possible results, I decided to select intervals by finding which notes would match if I constructed a circle of fifths using the ratio of 1.494412.i.e. (1.494412^2)/x.........to.....(1.494412^n)/x. x being an exponential of 2, the use of which as a divisor would give a result between 1 and 2 and hence in the first octave. Although I had originally intended to use MIDI pitch bend for the scale, I dreamt of an instrument which would be simple to play, in any key of up to 7 sharps or flats, for any competent guitarist.

The following morning my feet led me to Denmark St. where I casually mentioned that I had dreamt about this very strange guitar, and asked how I could make one. The only guitar I owned, liked, and was willing to modify was a Westone Raider II. I agreed a price with Graham Noden of Andy's, and rushed home to calculate the fret positions. I listed the note names, frequencies in Hertz, ratios, and fret positions for 20 frets per octave and a total of 35 frets; and delivered the Westone, instructions, and a detailed explanation of the mathematics to determine the fret positions.

On Christmas eve, Graham phoned me to say that he had started on the guitar, but that the ninth fret seemed to be in the wrong place, so would I please check my calculations. I did, and called him back with the position for an extra fret, between what had previously been the ninth and tenth frets.

When I called to pick up the finished job, Graham had already left for Christmas. I took home the first instrument to play the scale a renowned genius had first described two hundred and eleven years earlier. The tuning had to be done by ear, as only the open fifth string would be the same as any other guitar at 110 Hertz. I set it from a tuning fork, tuned the others from the appropriate frets, and slowly strummed an open G Major. The intervals sound foreign, but this chord certainly sings. Now for an open E; the same result. It works. Harrison must have imagined this moment when he wrote his book for some future generation to decipher. I spent the next two hours trying chords and melodies which I had previously only played on a conventional guitar. It was difficult at first to remember that sharp and flat notes must conform to the key of the piece, but with a little practice everything became playable except for the frets between the ninth and the octave, where something was definitely wrong. I checked the measurements. They matched my instructions exactly. Graham had certainly done his part perfectly. I checked the ratios against the fret positions; perfect up to the ninth, but between 10 and the octave nothing made sense. Two hours later I realised my mistake.

I had selected the wrong intervals, because my program had listed six fret positions on each line and after nine, I had counted the intervals out of synchronisation with the fret positions. Back into the computer. Two days later I had cracked it, and added a few extra frets on my instruction sheet. Lucy Guitar Mark V has twenty-five frets to the octave and a total of forty-five. It can be played in any key of up to 11 flats or 11 sharps.

Initially the layout of the Mark V looks confusing, for there are six pairs of close frets in each octave, and above the first octave, it was essential to use mandolin wire as the adjacent frets are very close. Although the guitar sounded harmonically perfect I found the average guitarist was initially intimidated by Mark V.

[Details and specifications of LucyTuned guitars]

This apprehension resulted in Lucy guitar Mark VII with nineteen frets to the octave by eliminating the least used of the frets in pairs. Mark VII is easier to play, but lacks the tonal versatility of Mark V, for it limits the number of keys which may be used to up to 7 flats or 9 sharps, or any conceivable key, if an error of (2L - 3s) = 14.367 cents between sharps and adjacent flats is tolerable. By compromising and replacing the pairs of frets on the Mark V by a single midpoint fret, this error may be reduced to 14.367/2 = 7.2 cents. Any pitch is sharpened or flattened by adding or subtracting the difference between a Large and a small interval i.e. (L-s) = 68.451 cents.

Applying the scale to synthesisers, I found that if the *black* keypads were all assigned to appropriate sharps or all to flat pitches, the system sounded consonant. Mixing sharps and flats caused interesting effects, but contradictory altered notes tended to sound dissonant.

To produce the most viable use on keyboard instruments of only twelve keypads per octave, the appropriate pitches for the *black* keys need to be programmed, or fast-loaded, from a selection of choices dependent upon the tonality of the piece to be played, but for some experimental keyboards of 31 or 53 keypads per octave the tunings may be fixed, and modulations between sharp keys and flat keys achieved.

LucyTuning using pitchbend and MIDI

This book is a constantly evolving document, to also report progress to readers. Many instruments, including harmonicas, banjos, basses etc. have now been produced, tuned or modified to this scale. The 19 frets per octave guitar has served its purpose as a starting point for timid players, but the limitations and tuning compromises soon became apparent. The 25 fret guitar has become the most versatile fretted instrument as all the notes are 'in tune', but this I'm sure will also evolve as musicians reach further into new tonalities. (as it has) As hundreds of people are now using these discoveries, research continues particularly into the connections to physics, mathematics, topology, and to music other than equal temperament for composition and performance.

Download Chapter One and associated files as Adobe (.pdf) format for view and print

|synopsis and order info. for this book| |Chapter Two| |Chapter Three| |To view table of more than sixty scales and their codes| |Recipe to build a physical model|

|LucyTuning homepage|

Other Tuning Systems

(other than LucyTuning)

Many tuning arrangements have been used in various cultures over the centuries. New systems are still being devised and experimented with, and all may be considered to fall into one or more of the categories below:

Pythagorean Tuning - (from cumulative steps of 3/2 and 4/3)

12tET breaks the octave into twelve equal semitones. Many integers other than 12 have been used in the last two centuries principally experimentally, and in the West.
Equal Temperaments some of which may also be considered as specific meantone tunings and mapped **as Large and small intervals**

Meantone tunings - (spirals of fourths and fifths)

Just Intonation - (small whole number frequency ratios)

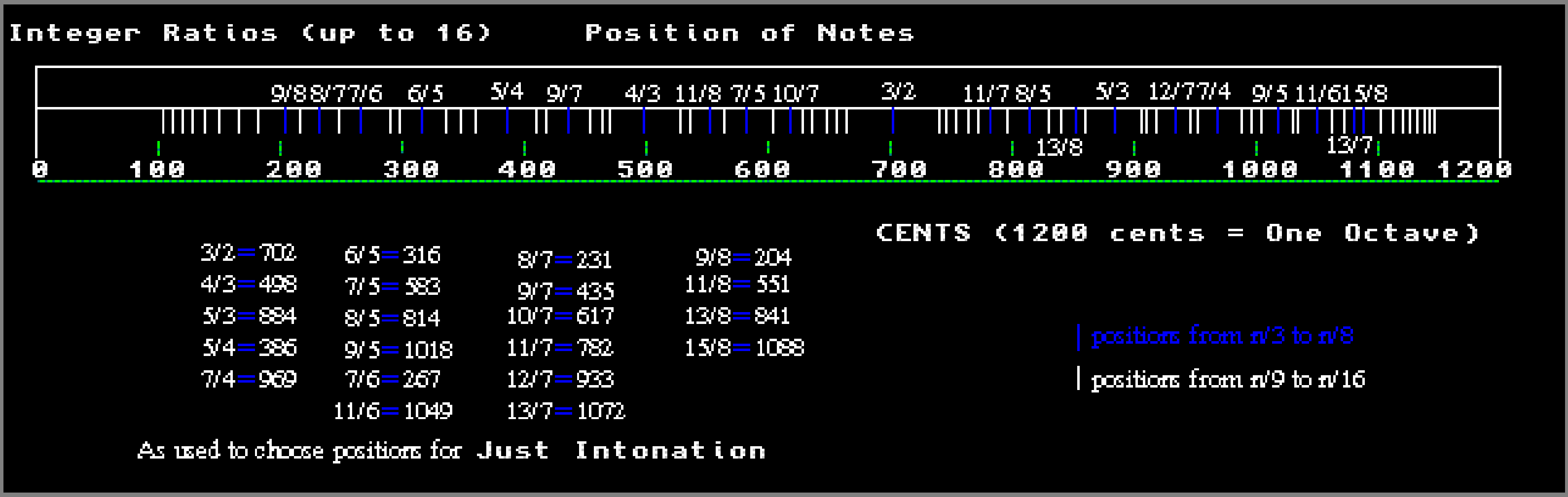
Tuning bibliography - (from mills.edu)

LucyTuning homepage

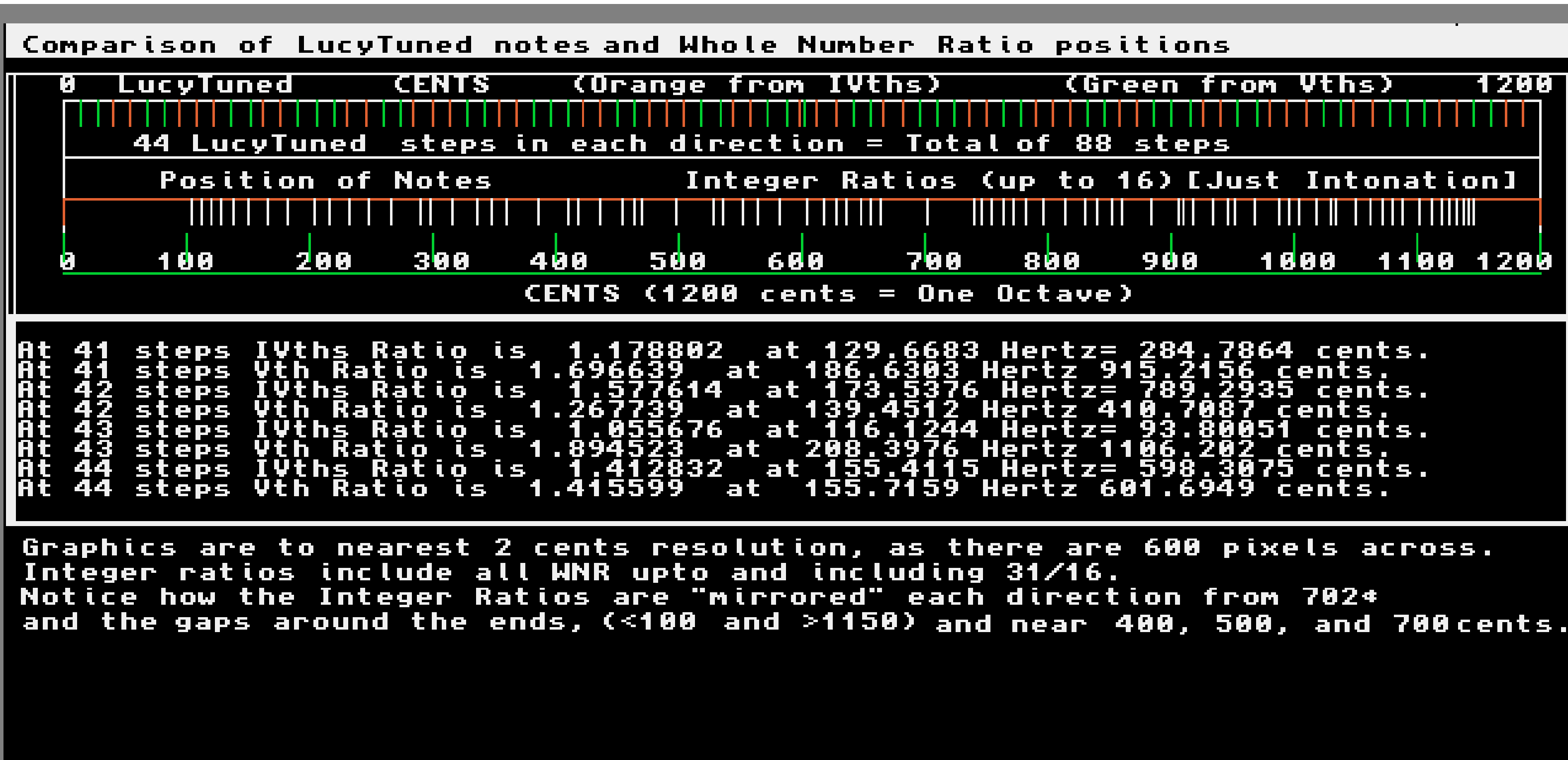
Just Intonation and whole number frequency ratio systems.

Just Intonation is a tuning system which operates on the assumption that musical harmonics only occur at frequencies which are at small integer ratios to the fundamental pitch. The ratios selected for Just Intonation seem to aim at producing "beatless" music; which to my mind is a futile, a folly. see *Pitch, Pi.....* Chapter One.

The diagram below shows where some of these ratios occur in the first octave. (i.e. 0 - 1200 cents)
The pattern shown begins with 3/2 at 702 cents, and ascends via the series 4/3, 5/3, 5/4, 6/5,...... .
The numbers 0 - 1200 in 100 cent increments are the positions used in 12tET as a reference. Remember that the graph represents cents. It is on a log scale, (like a fretboard, or slide rule), so that the midpoint in frequency (pitch*1.5 = 702 cents) is at the 58.5% (7.02/1200) position across the graph.
The positions shown in blue are the result of the integer ratios using all integers up to and including 8 for the divisor.
The positions shown in white use the integers from 9 to 16 as the divisor.



You may notice some **significant and interesting patterns** in this graph:
1. As the integers increase, many "new" values will fall at ratios which were previously hit. Eg. 6/4 = 3/2 etc.
2. There is an absence of "hits" close to values and their multiples, which are frequently hit. Notice the gaps around 3/2 (702 cents), 5/3 (884 cents) .... These are the ratios which are believed to be particularly significant in Just Intonation. If you are looking for a pitch as the Vth (i.e. around 700 cents) for your music; there is only one choice.
3. There are no hits near the ends of the graph, as to hit these locations requires large divisors.
4. The hits each side of 702 cents are "mirror images".
To compare graph above to first 44 steps of fourth and fifths using LucyTuning.



Many advocates of Just Intonation claim that lower integer ratios produce more consonant intervals, than higher integer ratios. The exact size of intervals is determined by their position in a scale. Eg. from C to D (One Large interval (L) in LucyTuning), may be of different sizes in Just Intonation, dependent upon what note is considered to be the tonic of the scale in which it is used. This can result in what is know as in JI circles, "wandering tonics"., and creates retuning requirements during modulation and transposition. I consider JI to be a simplistic, paradoxical, naive, single dimensional and static mapping system for tuning, although many "die-hards", are currently attempting to resuscitate it.

Four fourths plus one third: should they equal two octaves?

Note	Position	Just Intonation	Pythagorean Tuning	Interval L & s to next
C	I	4/3	4/3	2L+s
F	IV	4/3	4/3	2L+s
Bb	bVII	4/3	4/3	2L+s
Eb	bX	4/3	4/3	2L+s
Ab	bXIV	5/4	81/64	2L
C	XVI	Product is (256/81)*(5/4)=	Product is (256/81)*(81/64)=	

Total Ratio	2 Octaves 4.0	**Less than 2 Octaves** 3.9506173	2 Octaves 4.0	10L+4s 4.0
Note	Position	Just Intonation	Pythagorean Tuning	Interval L & s

Info. on Meantone tuning
Info on Pythagorean tuning
Info. on Equal temperaments

LucyTuning homepage

If you are still convinced that Just Intonation could be useful for your musical endeavours; here are some links to more information on other microtuning and associated sites so that you can see, hear, and draw your own conclusions.
Bookmark here first; and come back soon!.

Links out of LucyTuning to other websites about microtuning and other views on tuning.
Interesting ideas on timbre and tuning from Sethares **(recommended)**
American Festival of Microtonality
Southeast Just Intonation Center
FTP to Scala - a tuning program
RealTimeTuner
Bill Alves - JI information
Justonic commercial JI tuning program
Partch recordings
- bye-bye!

Pythagorean tuning is derived from the 3/2 frequency ratio, and has been believed by some to be the basis of many historical scales.

Pythagorean tuning uses steps of the frequency ratios **3/2** (702 cents) to generate steps of **fifths**; and **4/3** (498 cents) to generate steps of **fourths.** This results in a spiral tuning yet the resultant harmony is generally agreed to be *unsatisfactory*.

Some authorities claim that this is the basis of all Indian tunings, and that North Indian scales are derived from steps of fifths in the *sa* - *ma* pattern; whilst the South Indian tuning uses steps of fourths in the *sa* - *pa* pattern. As Indian musical practice tends to contain many ornamentations, much vibrato, and glissando, this is doubtful.

Steps of Fifths

Indian Name	Position	Western Equivalent	Ratio	Cents
Sa	I	C	2/1	1200.0
Ma	V	G	3/2	702.0
Kai. N	II	D	9/8	203.9
Sadh G	VI	A	27/16	905.9
Sudh D	III	E	81/64	407.8
Sudh R	VII	B	243/128	1109.8
Pra M	#IV	F#	729/512	611.7
Kak N	#I	C#	2187/2048	113.7
Antara G	#V	G#	6561/4096	815.6

Sadh G	#II	D#	19683/16384	317.6
Ch D	#VI	A#	59049/32768	1019.6
Ch R	#III	E#	177147/131072	521.5
Pa (a comma flatter)	#VII	B#	531441/524288	23.5

Steps of Fourths

Indian Name	Position	Western Equivalent	Ratio	Cents
Sa	I	C	2/1	1200.0
Pa	IV	F	4/3	498.0
Ch.R	bVII	Bb	16/9	996.1
Ch. D	bIII	Eb	32/27	294.1
Ant G	bVI	Ab	128/81	792.2
Kakali N	bII	Db	256/243	90.2
Pra M	bV	Gb	1024/729	588.3

Sud R	bVIII	Cb	4096/2187	1086.3
Sud D	bIV	Fb	8192/6561	384.4
Sadh G	bbVII	Bbb	32768/19683	882.4
Kaisiki N	bbIII	Ebb	65536/59049	180.5
Sud M	bbVI	Abb	262144/177147	678.5
Pa (a comma sharper)	bbII	Dbb	1048576/531441	1176.5

LucyTuning homepage

Compare to LucyTuned values (Pythagorean flats are flatter and sharps are sharper)

Equal temperaments

Meantone Tunings

Hybrid tunings mapped using L & s

Download Microsoft Works 4.0 Spreadsheet (includes adjustable cent values and fretting positions for Pythagorean Tuning)- worksfret.zip

Integer frequency ratio and Just Intonation tunings

back to *Pitch, Pi,....* Chapter One

Return to LucyTuning homepage

The unfortunate term ***Meantone*** seems to be derived from the traditional integer frequency ratio logic, as producing a spiral interval mapping from average deviations from integer ratios and expressed as commas.

Eg Quarter comma meantone.

Some of the systems are listed below:

Meantone tuning	derived from	Value of Fifth (in cents)	Large Interval (L)	small interval (s)
1/3 comma	6/5	694.77	189.58	126.15
LucyTuning	2(^(1/(2*pi))	695.49	190.99	122.54
Kornerup Phi	phi={(5^(1/2)+1)/2} L=(s*phi)	696.21	192.43	118.93
1/4 comma	5/4	696.58	193.16	117.10
Just 7/4 negative	7/4	696.88	193.76	115.60
Wilson's Meta-Meantone	many values	697.07	194.14	114.65
1/5 comma	15/8	697.65	195.30	111.75
12tET	1200/12	700	200	100
Helmholtz	5/4	701.71	203.42	91.45
Sabat-Garibaldi 1/9 Schisma	6/5	701.74	203.48	91.30
Pythagorean	3/2 and 4/3	701.96	203.91	90.23
7/4 Just	7/4	702.23	204.46	88.85

for more information on Pythagorean tuning and the anomalies and lemmas of this system and

Just Intonation and other historical integer frequency ratio tunings

Many equal temperaments and circular tunings could also be considered meantones, yet after a whole number of steps of fourths or fifths, they arrive at the same position at which they began.

For independent mathematical and graphic analyses of these tuning systems

Go to umich.edu for mathematical comparison of LucyTuning to 1/4 comma meantone and 12tET

Link to Tantrum (Mac program for temperament and tuning analysis and comparison)
More comparisons of LucyTuning to other meantone and equal temperaments.

Meantones are usually considered to be neutral (i.e. 700 cents for the fifth in 12tET); negative (less than 700 cents), or positive (more than 700 cents).

All meantones can be notated in conventional music notation. In negative meantones (like LucyTuning) the sharp of a note will be closer to the starting note than to the Large interval above; whereas in positive meantones it will be nearer to the Larger interval above. i.e in negative meantones L< 2s; in positive meantone L> 2s and in 12tET L=2s.

Return to Chapter One of Pitch, Pi, ...

Get Microsoft Works 4.0 adjustable spreadsheet of fretting positions, and cent + ratio values for any meantone and most xtET tunings. (worksfret.zip)

Return to LucyTuning homepage

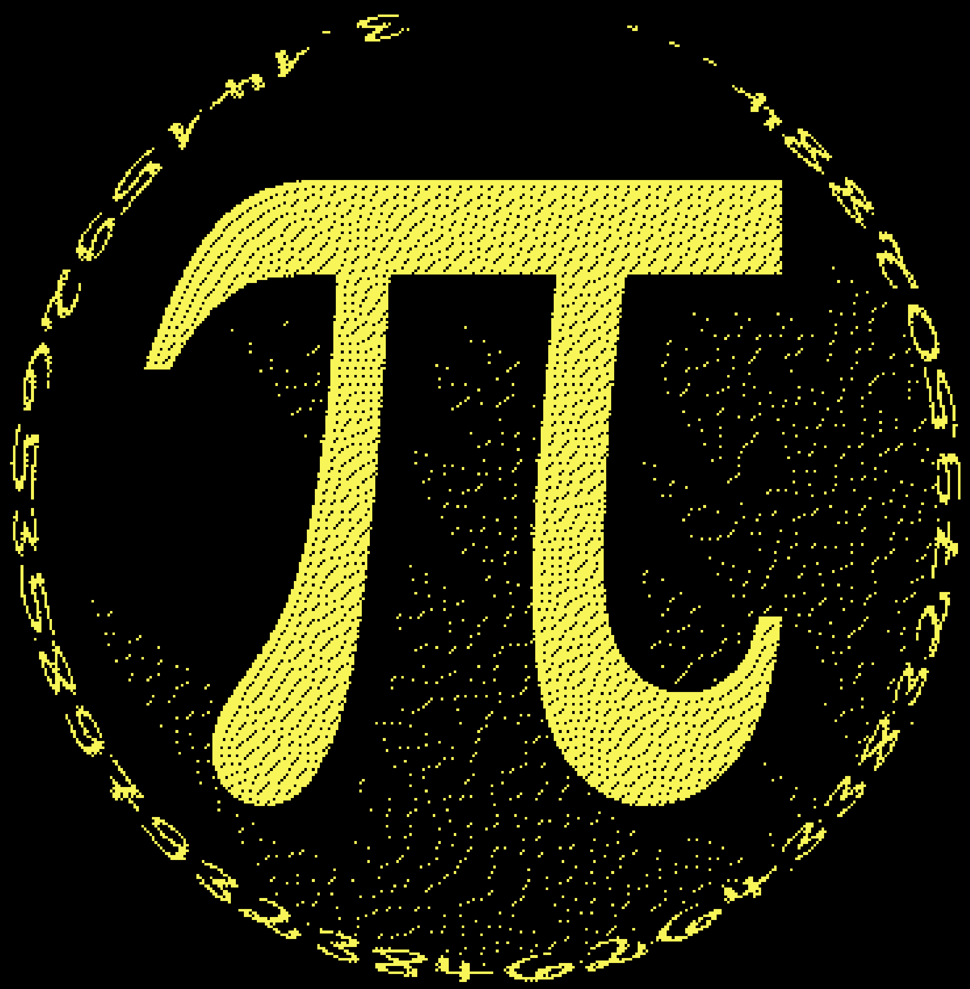
thoughts on:

π & φ

Pi and Phi

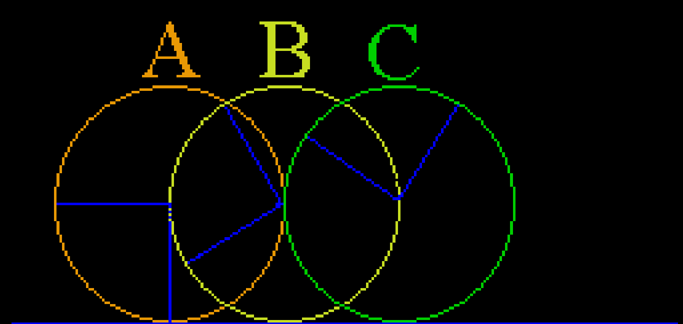
π Π

Pi is the ratio of the diameter to the circumference of a circle. It is usually represented by the Greek letter *pi*. There are many ways to derive the value and the calculation of ***pi*** has been refined during history from a simple 22/7 ratio to computer calculations running to many millions of digits. ***Pi*** is both irrational, (its infinitely long string of decimal places never repeats), and transcendental, (it is not a solution of any polynomial equation with whole number coefficients), occurs naturally and is used in all known cultures.



3.14159 26535 89793 23846 26433 83279 50288 41971 etc..

There are many ways to calculate this unique number.
For example
* By continuing the series 1 - 1/3 + 1/5 - 1/7 + 1/9 and multiplying the result by four gives pi.
* Adding the reciprocals of the squares (i.e. 1/1 + 1/4 + 1/9 + 1/9 + 1/16 + 1/25 ....) gives (pi^2)/6.



A circle rotated along the blue line from position A to position B turns thru $180/\pi^{o}$ degrees (i.e. one radian).
From A to C through two radians $= 360/\pi^{o}$.

ϕ Φ

Phi is also known as the *Golden* or *Divine Ratio*, and has been used in many models for patterns found in nature and artistic concepts. Mathematically, it is half of the square root of five, plus one....

$$(5^{(1/2)}+1)/2 = 1.618034.....$$

Phi may be derived from the Fibonacci series, as it is the ratio of increase from the series:

1,2,3,5,8,13,21,34,55,89,144............

Phi is the only number which is one greater that its reciprocal.

$$phi = 1 + (1/phi)$$

$$\text{which is } 1.618...... = 1 + (1/1.618.....)$$

π & φ

Pi and *Phi* are connected mathematically, for *pi* may be calculated from the Fibonacci sequence

The Fibonacci sequence is defined by the formula $u(0) = 0, u(1) = 1$
$u(n+1) = u(n) + u(n-1)$
So $u(2) = 1, u(3) = 2, u(4) = 5, u(5) = 8, u(6) = 13, u(7) = 21, u(8) = 34$.........

This connects to pi in the following manner:

Calculate all the Fibonacci numbers from $u(1)$ up to $u(m)$.
Multiply all the results together and call this $P(m)$.
Find the Least Common Multiple $LCM(m)$ from the numbers $u(1)$ through $u(m)$.

(The Least Common Multiple of two or more numbers is the smallest number that can be exactly divided by all the numbers)

Calculate the square root of *6logP(m)/logLCM(m)* and call it *Z(m)*.

As values of *m* increases the value of *Z(m)* becomes closer to ***pi.***

For *m=7 P(m)* is 1*1*2*3*5*8*13*21 = 32760
LCM(m)=10920 (i.e. 3*5*7*2*2*2*13)
6(log32760/log10920) = 6*4.515/4.038 = 6.709.* Square root is 2.590 = *Z(m)*

For *m=8 P(m)* is 1*1*2*3*5*8*13*21*34= 1113840
LCM(m)=185640
6(log1113840/log185640) = 6*6.0468/5.2687= 6.886.* Square root is 2.6241 = *Z(m)*

This serves to show the method for low numbers. As the values of *m* increase eventually *Z(m)* reaches ***pi***.

Back to LucyTuning homepage

LucySCALEDevelopments present:

LucyTuned Guitars

The first practical LucyTuned guitar was the Mk V, which was first made in 1986. It has twenty-five frets per octave.

The story of its design and construction can be found in [chapter one of *Pitch, Pi, and Other Musical Paradoxes*]. This is the design used by Arc-Angel and there are now a few hundred copies which have been made in all parts of the world. Although the design was patented, permission has been given for their non-commercial production for personal use. The design details have been evolving, and it is now proposed that new users start with the nineteen fret per octave model, and with experience add twelve further frets to make it thirty-one frets per octave.

Playing LucyTuned guitars

Diagram of LucyTuned and 12tET frettings

This design is intended to make the evolution from 12tET to LucyTuning as easy as possible for experienced musicians and new players. Although all frets, except the octave, are at different positions; the dots or marks found on the neck and fretboard of a conventional guitar are found at directly comparable positions on LucyTuned guitars. The familiar "landmarks" usually found at the 3, 5, 7, 9, and 12th frets on 12tET guitars, are placed at the bIII, IV, V, VI, and VIII positions. (i.e. C, D, E, and F# for the A (5th) string. This enables new users to use familiar open tunings and immediately navigate around the fretboard using familiar fingering.

When playing with more than 19 frets per octave you will use pairs of close frets. Placing your fingers **below** (towards the nut from) **the pair** will sound the **flatter** note. Playing **on the pair** will sound the **sharper** of the two notes.

You will notice that all the **sharper** of the notes sounded from the pairs will be in **sharp keys**; the **lower**

of the pair being in **flat keys**.

Tuning LucyTuned guitars

Any tuning of the open strings may be used: conventional (EADGBE), slack key, alternative etc. yet each string will need to be referenced to A4 = 440 Hz. and other notes fine tuned (+/- a few cents). This may be done by matching to frets on adjacent strings, using "harmonics", or an electronic tuner. The tuning needs to be very precise, yet when you have got correct, it will be very apparent, for chords you play will sound very "in tune".

Using conventional tuning the changes are:

Open String	**. . . 6 . E**	**. . 5 . A**	**. 4 . D**	**3 . G**	**. . . . . 2 . B**	**. . . 1 . E**
Change (cents)	4.5 cents **flat** (**b**)	**same** as 12 tET **110 Hz.**	**4.5** cents **sharp** (#)	**9.0** cents **sharp** (#)	**9.0** cents flat (**b**)	**4.5** cents flat (**b**)

Getting *your own* LucyTuned guitar

There are a number of options.

New Neck and fretboard

New necks can be manufactured for most solid guitars with any specified fretting by: John Carruthers, 346, Sunset Ave, Venice, California 90291. Phone1 (213) 392-3910 contact Jim Hetal.

Magnetic Fretboard Kits

Mark Rankin, (last seen in Phoenix, AZ.), mail at: Franklin City, Greensbackville, VA 23356, phone contact numbers 1 (714) 688-9894 and 1 (415) 658-1889 can provide a kit for interchangeable fretboards, which are held in position by a magnetic laminate.

DIY: You can produce your own, using magnetic laminate available from: Magna Visual Inc. 9400, Watson Road, St. Louis, Missouri 63126-1598: Voice 1 (314) 843-9000 or Fax 1 (314) 843-0000. ($15 for two sheets .045" x 12" x 24"). You will need to remove all frets and plane or sand down your fretboard to glue on a thin metal sheet (I have used thin galvanised roofing material), then cut the laminate to size, and attach the frets to the laminate.

You can then use the guitar fretless, or with a variety of interchangeable fretting systems.

Refretting

You can get an existing guitar refretted by any competent luthier. I use and recommend: Colin Noden at Andy's Guitar Shop, Denmark St. London W1. He is very experienced and usually busy, yet does an excellent job. Most luthiers will charge a couple of hundred dollars for the work, and will need the exact fret positions which depend upon your nut to bridge distance.
In the US, you might also try Glen Peterson.
If your instrument has other than 650 mm from nut to bridge, you will need to pro-rata the distances, or EMail to Charles Lucy from here. (lucy@harmonics.com) with your nut to bridge distance (in inches or millimeters) to get the AmigaBasic or spreadsheet program or a file of the output.

DIY: Doing it yourself is the least expensive route. Remove all the frets. Fill the holes with Plastic Wood. Allow to dry overnight. Sand the board and stick masking tape over it so that you can mark out the fret positions. Draw a straight line from the centre of the nut to the centre of the bridge as a reference for fret alignment, and mark each fret position. Cut fret grooves; remove the masking tape, insert the frets; secure them; trim; file; dress; set up guitar and enjoy playing your LucyTuned guitar.

I suggest 19 frets per octave initially, so that you can add more frets later as you gain playing experience. Use mandolin fretwire for the second octave, so that there will be space for the extra frets later.

Fret positions for LucyTuned 19 & 31 frets per octave instruments.

Intervals	Ratio	Cents
Large (L)	1.116633	190.9858

small (s)	1.073344	122.5354

Distance from Nut to Bridge = **650** (millimetres)

(for other other nut to bridge distances, values can be pro-rated)

Diagram of LucyTuned and12tET frettings

Note Name Guitar Fifth String (* = marks)	Scale Position	Distance Nut to fret First Octave	Fret No. of **19** for **First** **octave** (0-19)	Fret No. of **31** for **First** **octave** (0-31)	Large (L) and small (s) Intervals from nut. Add (5L+2s) for second octave	Distance Nut to fret Second Octave	Fret No. of **19** for **Second** **octave** (19-38)	Fret No. of **31** for **Second** **octave** (31-62)
A	I	-	Nut	Nut	Zero & 5L+2s	325.0	19	31
Bbb	bbII	019.9923	-	1	2s-L	334.9962	-	32
A#	#I	025.1987	1	2	L-s	337.5994	20	33
Bb	bII	044.4160	2	3	s	347.2080	21	34
Ax	xI	049.4295	-	4	2L-2s	349.7103	-	35
B	II	067.8928	3	5	L	358.9464	22	36
Cb	bbIII	085.7970	-	6	2s	367.8985	-	37
B#	#II	090.4595	4	7	2L-s	370.2298	23	38
*C	bIII*	107.6696	*5	8*	L+s	378.8348	*24	39*
Dbb	bbIV	124.3503	-	9	3s	387.1751	-	40
C#	III	128.6942	6	10	2L	389.3471	25	41
Db	bIV	144.7283	7	11	L+2s	397.3641	26	42
Cx	#III	148.9038	-	12	3L-s	399.4519	-	43
*D	IV*	164.3163	*8	13*	2L+s	407.1581	*27	44*
Ebb	bbV	179.2546	-	14	L+3s	414.6273	-	45
D#	#IV	183.1449	9	15	3L	416.5724	28	46
Eb	bV	197.5042	10	16	2L+2s	423.7521	29	47

Dx	xIV	201.2436	-	17	4L-s	425.6218	-	48
*E	V*	215.9462	*11	18*	3L+s	432.5231	*30	49*
Fb	bbVI	228.4242	-	19	2L+3s	439.2121	-	50
E#	#V	231.9081	12	20	4L	440.9541	31	51
F	bVI	244.7676	13	21	3L+2s	447.3838	32	52
Ex	xV	248.1164	-	22	5L-s	449.0582	-	53
*F#	VI*	260.4773	*14	23*	4L+s	455.2387	*33	54*
Gb	bbVII	272.4580	15	24	3L+3s	461.2290	34	55
Fx	#VI	275.5781	-	25	5L	462.7890	-	56
G	bVII	287.0943	16	26	4L+2s	468.5472	35	57
Abb	bbVIII	298.2564	-	27	3L+4s	474.1282	-	58
G#	VII	301.1632	17	28	5L+s	475.5816	36	59
Ab	bVIII	311.8925	18	29	4L+3s	480.9462	37	60
Gx	#VII	314.6866	-	30	6L	482.3433	-	61
**A	VIII**	325.0000	19**	31**	5L+2s	487.5000	38**	62**

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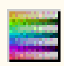
THE-X-PERIENCE TOP 100!

101 GuitarSites.com (links to other guitar sites)



Alternative Open Guitar and Bass tunings

LucyTuning homepage

Using MIDI Pitchbend and MIDI Tuning Dump to LucyTune your music.

Pitchbend

It is possible to LucyTuned your sequences to the nearest 64th of a semitone using MIDI pitchbend.
Check that your MIDI equipment can recognise and respond to pitchbend data
The current pitchbend range operating on your equipment may be adjustable (see the operating manual), if this is not clear or you wish to test the current range test it as follows:

1. Make a simple MIDI sequence of two adjacent notes (one semitone apart) Eg. G#4 and A4. Play the sequence and you should hear two notes a semitone apart.

2. Now add a pitchbend value to the lower of the two notes, at a value of +4096 and a value of 0 to the higher note. Listen to the two notes again. If the notes are now the same pitch your pitchbend range is set at 4096 units per semitone.

Experiment by changing the values until you get it right, and discover how many pitchbend units are required to bend your equipment by one semitone. This will tell you which column (*d*, *or e*) of the table below to use.

Pitchbend usually effects only the assigned channel, and all notes played on that channel will be "bent" until another pitchbend command is received. Therefore to LucyTune your sequence you may have to move notes to other channels, so that there is only ever one note per channel at any time.

(Remember to reset all the A's to zero)

3. Each note will need to be pitchbent by the appropriate amount. This is a tedious procedure, yet the results can be quite satisfying. You can make the conversion easier by using copy and paste in your sequencer's edit program.

Using the table below and the appropriate pitchbend ranges (columns *d, or e)* you can set the pitchbend of each note (column **a**) which you wish to use by adjusting the value for the MIDI note named in column *b.*

The cent values are also included for users of Ensoniq, Korg M series, and other cent programmable equipment (column c*).*

Lucy **Note** Name **a**	**Bend** 12tET Note *b*	**cents** from A4=440Hz c	**Bend** +/- 64ths *d*	**Bend** 4096 ths *e*	**Ratio from A=1.0000** *f*	**Dump** 64ths (Proteus) **g**	**Dump** xx (hex) semitone **h**	**Dump** yy (hex) MSB **i**	**Dump** zz (hex) LSB **j**
C	C	0313.521	+9	+554	1.198531	0009	00	11	27
Dbb	C	0367.605	+43	+2769	1.236564	0043	00	56	45
C#	C#	0381.972	-12	-738	1.246869	0052	00	68	75
Db	C#	0436.056	+23	+1477	1.286437	0087	01	2E	14
Cx	D	0450.424	-32	-2031	1.297156	0096	01	40	45
D	D	0504.507	+3	+185	1.338320	0131	02	05	62
Ebb	D	0558.591	+37	+2400	1.380789	0166	02	4B	00
D#	D#	0572.958	-17	-1108	1.392295	0175	02	5D	31
Eb	D#	0627.042	+17	+1108	1.436478	0209	03	22	4F
Dx	E	0641.410	-37	-2400	1.448445	0218	03	33	00
E	E	0695.493	-3	-185	1.494412	0253	03	7A	1E
Fb	E	0749.577	+32	+2031	1.541834	0288	04	3F	3B
E#	F	0763.944	-23	-1477	1.554682	0297	04	51	6C
F	F	0818.028	+12	+738	1.604018	0332	05	17	0A
Gbb	F	0872.112	+46	+2954	1.654918	0366	05	5C	27
F#	F#	0886.479	-9	-554	1.668709	0375	05	6E	59
Gb	F#	0940.563	+26	+1661	1.721663	0410	06	33	76
Fx	G	0954.931	-29	-1846	1.736018	0419	06	46	28
G	G	1009.014	+6	+369	1.791099	0454	07	0B	45
Abb	G	1063.098	+40	+2584	1.847936	0488	07	50	62
G#	G#	1077.465	-14	-923	1.863336	0498	07	63	14
Ab	G#	1131.549	+20	+1292	1.922466	0532	08	28	31
Gx	A	1145.917	-35	-2215	1.938491	0542	08	3A	63
A	A	00000.000	0	0	1.000000	0576	09	00	00
Bbb	A	0054.083	+35	+2215	1.031734	0611	09	45	1D
A#	A#	0068.451	-20	-1292	1.040331	0620	09	57	4F

Bb	A#	0122.535	+14	+923	1.073344	0654	0A	1C	6C
Ax	B	0136.903	-40	-2584	1.082291	0664	0A	2F	1E
B	B	0190.986	-6	-369	1.116633	0698	0A	74	3B
Cb	B	0245.070	+29	+1846	1.152068	0733	0B	39	58
B#	C	0259.438	-26	-1661	1.161667	0742	0B	4C	0A

Programs to microtune your MIDI files using pitchbend

Oct 1999 A recent trio of PC programs called Midi Tempering Utilities by Fred Nachbaur (free midi pitchbend software) can be used to midi microtune existing midi files.

Download lucy**.dat files for use with Nachbaur miditemp program.**

Lucy0f5s 0 flats - 5 sharps (i.e. black notes are C#-D#-F#-G#-A#)

Lucy1f4s 1 flat - 4 sharps (i.e. black notes are C#-D#-F#-G#-Bb)

Lucy2f3s 2 flats - 3 sharps (i.e. black notes are C#-Eb-F#-G#-Bb)

Lucy3f2s 3 flats - 2 sharps (i.e. black notes are C#-Eb-F#-Ab-Bb)

Lucy4f1s 4 flats - 1 sharp (i.e. black notes are Db-Eb-F#-Ab-Bb)

Lucy5f0s 5 flats - 0 sharps (i.e. black notes are Db-Eb-Gb-Ab-Bb)

To LucyTune your files with these programs use the ratios from A = 1.000000 in the *g* column above

MIDI Tuning Dump

In 1992, a new MIDI tuning dump standard was introduced. This is intended to transmit tuning data to a resolution of 16,384 units per semitone, (196,608 per octave). As many of the psychological effects of LucyTuning depend upon subsonic beating: the more accurate the tuning; the greater the effect. Unfortunately I have yet to find a manufacturer who has fully implemented this standard, although it should eventually happen. (Please encourage manufacturers to introduce it in their new products). A number of tuning programs (eg. Tuning Wrench) and some hardware (eg. Proteus 3) already use these values to transfer tuning data.

By using the table above in the Dump columns (**g, h, i, and j**), you can tune to the LucyTuned notes listed in column **a.** The tuning resolution which is played will depend upon your hardware. The 64th of a semitone (Proteus) and xx (hex) values shown in the table are for the lowest octave. For higher octaves add 768 units per octave to the 64th of a semitone (Proteus) column (**g**), or 12 (0C in hex) per octave to the xx column (**h**).

[Frequency data format (all bytes in hex)]

xx semitone = 100 cent units; yy MSB (Most Significant Byte) of fraction (1/128 semitone) = 0.78125 cent units; LSB (Least Significant Byte) of fraction (1/16384 semitone) = 0.0061 cent units

LucyTuning table of 52 arrangements for microtunable synths and samplers

Brian Pugley's EMagic Logic **Tuning** Environments include LucyTuning features - download from here.
Download printable version of this page (Adobe .pdf)

Back to LucyTuning homepage

an extract from:

Pitch, Pi, and Other Musical Paradoxes (A Practical Guide to Natural Microtonality)

by Charles E. H. Lucy ISBN 0-9512879-0-7

Chapter One.

IS THIS THE LOST MUSIC OF THE SPHERES?

After twenty five years of playing, I realised that I was still unable to tune any guitar so that it sounded in tune for both an open G major and an open E major. I tried tuning forks, pitch pipes, electronic tuners and harmonics at the seventh, fifth and twelfth frets. I sampled the best guitars I could find, but none would "sing" for both chords. At the risk of appearing tone deaf, I confessed my incompetence to a few trusted musical friends, secretly hoping that someone would initiate me into the secret of tuning guitars, so that they would "sing" for all chords. A few admitted that they had the same problem, but none would reveal the secret.

I had read how the position of the frets was calculated from the twelfth root of two, so that each of the twelve semitones in an octave had equal intervals of 100 cents. It is the equality of these intervals, which allows us to easily modulate or transpose into any of the twelve keys.

Without realising it, I had begun a quest for the lost music of the spheres. I still secretly doubted my musical ear, but rationalised my search by professing an interest in microtonal music, and claimed to be searching for the next step in the evolution of music. First stop library, where I found Helmholtz's book *On The Sensations Of Tone*. He introduced me to thousands of alternative tunings. The only universal 'truth', on which all tunings agreed was that halving the length of a string doubles the frequency, and produces a ratio of 2.00000:1 which is known as an octave. This octave was subdivided in three basic ways;

1) By a geometric progression, with any number of equal intervals.Eg. 12 (as on conventional guitars) [100 cents per semitone or interval]; 31 as advocated by Huyghens (1629-1695) [38.71 cents per interval], or 53 by Mercator and Bosanquet (1876 Treatise) [22.64 cents];

2) By low whole number ratios. Eg. (Just Intonation) 3:2 for the Vth; 5:4 for the major third etc.

3) By cumulative fifths. Eg. Pythagorean Tuning 3:2 for the Vth; but 81:64 for the major third.

There are also hybrids of the other three, Eg. Meantone Temperaments .

If the only point of agreement is that the octave ratio should be exactly two; how do we explain the phenomenon of stretched octaves, used by some piano tuners?

The Pythagorean system , instead of arriving at the octave after 12 steps i.e. (3^12)/(2^18) becomes 531441/262144 = 2.0272865 instead of 2.0000 as we assume when tuning a guitar by fifths at the seventh fret. This difference or error is known as *Pythagoras' lemma*. Every system seems to be an imperfect compromise, which is probably why mathematicians and musicians have devoted millions of hours to searching for the perfect scale.

Initially, the idea of 53 notes on a geometric progression seemed to be a sensible solution, and I read that

Bosanquet's harmonium on this scale had been in the Kensington Science Museum since the 1880's. I went to find it, but it was in storage. Instead I found Mr. Chew. I told him of my quest and that I had a hunch that the solution was in some way connected with the music of the spheres and the Greek letter " Pi ".

π

"That's what Harrison thought."

I enquired further and discovered that **John Harrison** ***(1693-1776)***, an horologist, had discovered longitude and won a £20,000 prize from Parliament after the personal intervention of George III. I was directed to the Clockmakers' Library in the Guildhall, and there found a treasured copy of *A Description concerning such Mechanism as will afford a nice, or true Mensuration of Time; together with Some Account of the Attempts for the Discovery of the Longitude by the Moon; and also An Account of the Discovery of the Scale of Musick*, harrison.zip only to be refused permission to photocopy any of it, due to its antiquity. I eventually acquired photocopies of the relevant pages from another source. Harrison had written in such an obscure style that I suspected that he intended to hide its secrets from all but the most diligent enquirer.

The essence of what Harrison said is as follows:

"The *natural scale of music* is associated with the ratio of the diameter of a circle to its circumference." (i.e. pi = 3.14159265358979323846 etc.)

"This scale is based on two intervals;"

1) The *Larger note* as he calls it; This is a ratio of 2 to the 2*pi root of 2, or in BASIC computer terms 2^(1/(2*pi)), which equals a ratio of 1.116633 or 190.9858 cents, approximately 1.91 frets on a conventional guitar. (L)

2) The *lesser note*, which is half the difference between five *Larger notes* (5L) and an octave. i.e.(2/(2^(1/(2*pi)))^5)^(1/2), giving a ratio of 1.073344 or 122.5354 cents, an interval of approx. 1.23 frets. (s)

The equivalent of the fifth (i.e. seventh fret on guitar) is composed of three Large (3L) plus one small note (s) i.e. (3L+s) = (190.986*3) + (122.535) = 695.493 cents or ratio of 1.494412.

The equivalent of the fourth (IV) (fifth fret) is 2L+s = 504.507 cents.

Harrison discovered this scale experimenting with monochords and a viol, and trained a church choir in Lincolnshire to sing it.

From this information I constructed a number of computer programs to explore all the possible permutations and combinations which could generate a practical scale. My problem was to decide which of the hundreds of intervals which I calculated by addition and subtraction of multiples of the Large (L) and small (s) interval should be used. Faced with pages of possible results, I decided to select intervals by finding which notes would match if I constructed a circle of fifths using the ratio of 1.494412.i.e. (1.494412^2)/x.........to.....(1.494412^n)/x. x being an exponential of 2, the use of which as a divisor would give a result between 1 and 2 and hence in the first octave. Although I had originally intended to use MIDI pitch bend for the scale, I dreamt of an instrument which would be simple to play, in any key of up to 7 sharps or flats, for any competent guitarist.

The following morning my feet led me to Denmark St. where I casually mentioned that I had dreamt about this very strange guitar, and asked how I could make one. The only guitar I owned, liked, and was willing to modify was a Westone Raider II. I agreed a price with Graham Noden of Andy's, and rushed home to calculate the fret positions. I listed the note names, frequencies in Hertz, ratios, and fret positions for 20 frets per octave and a total of 35 frets; and delivered the Westone, instructions, and a detailed explanation of the mathematics to determine the fret positions.

On Christmas eve, Graham phoned me to say that he had started on the guitar, but that the ninth fret seemed to be in the wrong place, so would I please check my calculations. I did, and called him back with the position for an extra fret, between what had previously been the ninth and tenth frets.

When I called to pick up the finished job, Graham had already left for Christmas. I took home the first instrument to play the scale a renowned genius had first described two hundred and eleven years earlier. The tuning had to be done by ear, as only the open fifth string would be the same as any other guitar at 110 Hertz. I set it from a tuning fork, tuned the others from the appropriate frets, and slowly strummed an open G Major. The intervals sound foreign, but this chord certainly sings. Now for an open E; the same result. It works. Harrison must have imagined this moment when he wrote his book for some future generation to decipher. I spent the next two hours trying chords and melodies which I had previously only played on a conventional guitar. It was difficult at first to remember that sharp and flat notes must conform to the key of the piece, but with a little practice everything became playable except for the frets between the ninth and the octave, where something was definitely wrong. I checked the measurements. They matched my instructions exactly. Graham had certainly done his part perfectly. I checked the ratios against the fret positions; perfect up to the ninth, but between 10 and the octave nothing made sense. Two hours later I realised my mistake.

I had selected the wrong intervals, because my program had listed six fret positions on each line and after nine, I had counted the intervals out of synchronisation with the fret positions. Back into the computer. Two days later I had cracked it, and added a few extra frets on my instruction sheet. Lucy Guitar Mark V has twenty-five frets to the octave and a total of forty-five. It can be played in any key of up to 11 flats or 11 sharps.

Initially the layout of the Mark V looks confusing, for there are six pairs of close frets in each octave, and above the first octave, it was essential to use mandolin wire as the adjacent frets are very close. Although the guitar sounded harmonically perfect I found the average guitarist was initially intimidated by Mark V.

Details and specifications of LucyTuned guitars

This apprehension resulted in Lucy guitar Mark VII with nineteen frets to the octave by eliminating the least used of the frets in pairs. Mark VII is easier to play, but lacks the tonal versatility of Mark V, for it limits the number of keys which may be used to up to 7 flats or 9 sharps, or any conceivable key, if an error of (2L - 3s) = 14.367 cents between sharps and adjacent flats is tolerable. By compromising and replacing the pairs of frets on the Mark V by a single midpoint fret, this error may be reduced to 14.367/2 = 7.2 cents. Any pitch is sharpened or flattened by adding or subtracting the difference between a Large and a small interval i.e. (L-s) = 68.451 cents.

Applying the scale to synthesisers, I found that if the *black* keypads were all assigned to appropriate sharps or all to flat pitches, the system sounded consonant. Mixing sharps and flats caused interesting effects, but contradictory altered notes tended to sound dissonant.

To produce the most viable use on keyboard instruments of only twelve keypads per octave, the appropriate pitches for the *black* keys need to be programmed, or fast-loaded, from a selection of choices dependent upon the tonality of the piece to be played, but for some experimental keyboards of 31 or 53 keypads per octave the tunings may be fixed, and modulations between sharp keys and flat keys achieved.

LucyTuning using pitchbend and MIDI

This book is a constantly evolving document, to also report progress to readers. Many instruments, including harmonicas, banjos, basses etc. have now been produced, tuned or modified to this scale. The 19 frets per octave guitar has served its purpose as a starting point for timid players, but the limitations and tuning compromises soon became apparent. The 25 fret guitar has become the most versatile fretted instrument as all the notes are 'in tune', but this I'm sure will also evolve as musicians reach further into new tonalities. (as it has) As hundreds of people are now using these discoveries, research continues particularly into the connections to physics, mathematics, topology, and to music other than equal temperament for composition and performance.

|synopsis and order info. for this book| |Chapter Two| |Chapter Three| |To view table of more than sixty scales and their codes| |Recipe to build a physical model|

|LucyTuning homepage|

Other Tuning Systems

(other than LucyTuning)

Many tuning arrangements have been used in various cultures over the centuries. New systems are still being devised and experimented with, and all may be considered to fall into one or more of the categories below:

Pythagorean Tuning - (from cumulative steps of 3/2 and 4/3)

12tET breaks the octave into twelve equal semitones. Many integers other than 12 have been used in the last two centuries principally experimentally, and in the West.
Equal Temperaments some of which may also be considered as specific meantone tunings and mapped **as Large and small intervals**

Meantone tunings - (spirals of fourths and fifths)

Just Intonation - (small whole number frequency ratios)

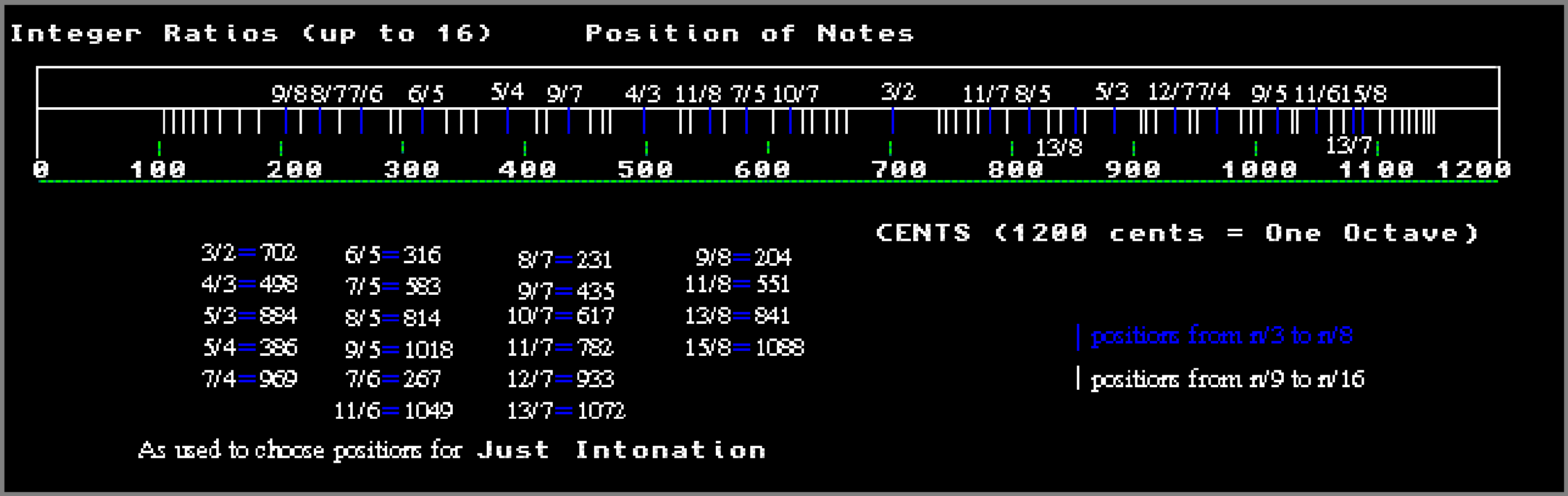
Tuning bibliography - (from mills.edu)

LucyTuning homepage

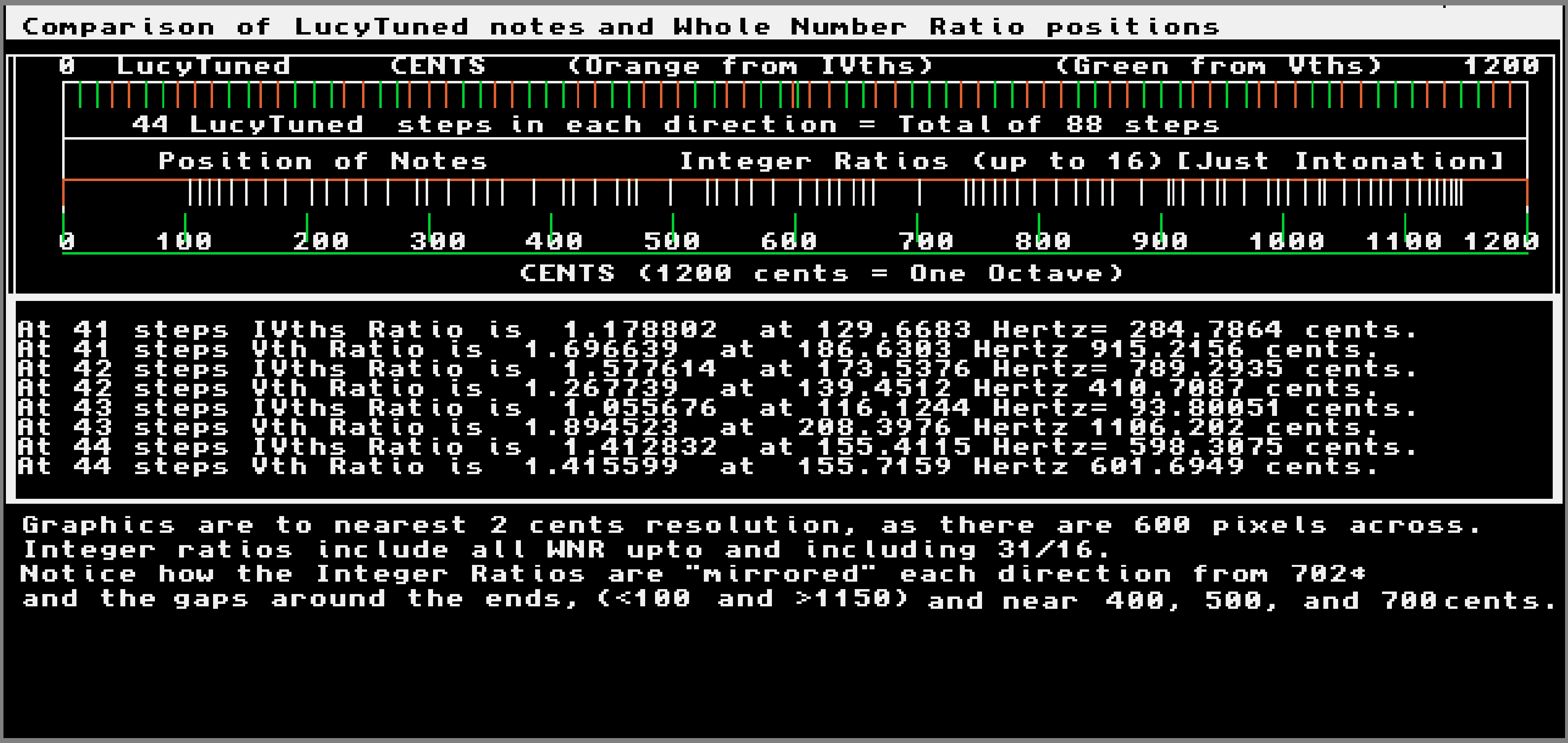
Just Intonation and whole number frequency ratio systems.

Just Intonation is a tuning system which operates on the assumption that musical harmonics only occur at frequencies which are at small integer ratios to the fundamental pitch. The ratios selected for Just Intonation seem to aim at producing "beatless" music; which to my mind is a futile, a folly. see *Pitch, Pi.....* Chapter One.

The diagram below shows where some of these ratios occur in the first octave. (i.e. 0 - 1200 cents)
The pattern shown begins with 3/2 at 702 cents, and ascends via the series 4/3, 5/3, 5/4, 6/5,...... .
The numbers 0 - 1200 in 100 cent increments are the positions used in 12tET as a reference. Remember that the graph represents cents. It is on a log scale, (like a fretboard, or slide rule), so that the midpoint in frequency (pitch*1.5 = 702 cents) is at the 58.5% (7.02/1200) position across the graph.
The positions shown in blue are the result of the integer ratios using all integers up to and including 8 for the divisor.
The positions shown in white use the integers from 9 to 16 as the divisor.



You may notice some **significant and interesting patterns** in this graph:
1. As the integers increase, many "new" values will fall at ratios which were previously hit. Eg. 6/4 = 3/2 etc.
2. There is an absence of "hits" close to values and their multiples, which are frequently hit. Notice the gaps around 3/2 (702 cents), 5/3 (884 cents) .... These are the ratios which are believed to be particularly significant in Just Intonation. If you are looking for a pitch as the Vth (i.e. around 700 cents) for your music; there is only one choice.
3. There are no hits near the ends of the graph, as to hit these locations requires large divisors.
4. The hits each side of 702 cents are "mirror images".
To compare graph above to first 44 steps of fourth and fifths using LucyTuning.



Many advocates of Just Intonation claim that lower integer ratios produce more consonant intervals, than higher integer ratios. The exact size of intervals is determined by their position in a scale. Eg. from C to D (One Large interval (L) in LucyTuning), may be of different sizes in Just Intonation, dependent upon what note is considered to be the tonic of the scale in which it is used. This can result in what is know as in JI circles, "wandering tonics"., and creates retuning requirements during modulation and transposition. I consider JI to be a simplistic, paradoxical, naive, single dimensional and static mapping system for tuning, although many "die-hards", are currently attempting to resuscitate it.

Four fourths plus one third: should they equal two octaves?

Note	Position	Just Intonation	Pythagorean Tuning	Interval L & s to next
C	I	4/3	4/3	2L+s
F	IV	4/3	4/3	2L+s
Bb	bVII	4/3	4/3	2L+s
Eb	bX	4/3	4/3	2L+s
Ab	bXIV	5/4	81/64	2L
C	XVI	Product is (256/81)*(5/4)=	Product is (256/81)*(81/64)=	
Total Ratio	2 Octaves 4.0	**Less than 2 Octaves** 3.9506173	2 Octaves 4.0	10L+4s 4.0
Note	Position	Just Intonation	Pythagorean Tuning	Interval L & s

Info. on Meantone tuning
Info on Pythagorean tuning

Info. on Equal temperaments

LucyTuning homepage

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ftp to Tuning ListArchive

If you are still convinced that Just Intonation could be useful for your musical endeavours; here are some links to more information on other microtuning and associated sites so that you can see, hear, and draw your own conclusions.

Bookmark here first; and come back soon!.

Links out of LucyTuning to other websites about microtuning and other views on tuning.

Interesting ideas on timbre and tuning from Sethares **(recommended)**

American Festival of Microtonality

Southeast Just Intonation Center

FTP to Scala - a tuning program

RealTimeTuner

Bill Alves - JI information

Justonic commercial JI tuning program

Partch recordings

Benjamin Tubb's music theory pages - Joseph Schillinger to Albert Silverman

- bye-bye!

Pythagorean tuning is derived from the 3/2 frequency ratio, and has been believed by some to be the basis of many historical scales.

Pythagorean tuning uses steps of the frequency ratios **3/2** (702 cents) to generate steps of **fifths**; and **4/3** (498 cents) to generate steps of **fourths.** This results in a spiral tuning yet the resultant harmony is generally agreed to be *unsatisfactory*.

Some authorities claim that this is the basis of all Indian tunings, and that North Indian scales are derived from steps of fifths in the *sa* - *ma* pattern; whilst the South Indian tuning uses steps of fourths in the *sa* - *pa* pattern. As Indian musical practice tends to contain many ornamentations, much vibrato, and glissando, this is doubtful.

Steps of Fifths

Indian Name	Position	Western Equivalent	Ratio	Cents
Sa	I	C	2/1	1200.0
Ma	V	G	3/2	702.0
Kai. N	II	D	9/8	203.9
Sadh G	VI	A	27/16	905.9
Sudh D	III	E	81/64	407.8
Sudh R	VII	B	243/128	1109.8
Pra M	#IV	F#	729/512	611.7
Kak N	#I	C#	2187/2048	113.7
Antara G	#V	G#	6561/4096	815.6
Sadh G	#II	D#	19683/16384	317.6
Ch D	#VI	A#	59049/32768	1019.6
Ch R	#III	E#	177147/131072	521.5
Pa (a comma flatter)	#VII	B#	531441/524288	23.5

Steps of Fourths

Indian Name	Position	Western Equivalent	Ratio	Cents
Sa	I	C	2/1	1200.0
Pa	IV	F	4/3	498.0
Ch.R	bVII	Bb	16/9	996.1
Ch. D	bIII	Eb	32/27	294.1
Ant G	bVI	Ab	128/81	792.2

Kakali N	bII	Db	256/243	90.2
Pra M	bV	Gb	1024/729	588.3
Sud R	bVIII	Cb	4096/2187	1086.3
Sud D	bIV	Fb	8192/6561	384.4
Sadh G	bbVII	Bbb	32768/19683	882.4
Kaisiki N	bbIII	Ebb	65536/59049	180.5
Sud M	bbVI	Abb	262144/177147	678.5
Pa (a comma sharper)	bbII	Dbb	1048576/531441	1176.5

LucyTuning homepage

Compare to LucyTuned values (Pythagorean flats are flatter and sharps are sharper)

Equal temperaments

Meantone Tunings

Hybrid tunings mapped using L & s

Download Microsoft Works 4.0 Spreadsheet (includes adjustable cent values and fretting positions for Pythagorean Tuning)- worksfret.zip

Integer frequency ratio and Just Intonation tunings

back to *Pitch, Pi*,..... Chapter One

Return to LucyTuning homepage

The unfortunate term ***Meantone*** seems to be derived from the traditional integer frequency ratio logic, as producing a spiral interval mapping from average deviations from integer ratios and expressed as commas.

Eg Quarter comma meantone.

Some of the systems are listed below:

Meantone tuning	derived from	Value of Fifth (in cents)	Large Interval (L)	small interval (s)
1/3 comma	6/5	694.77	189.58	126.15
LucyTuning	2(^(1/(2*pi))	695.49	190.99	122.54
Kornerup Phi	phi={(5^(1/2)+1)/2} L=(s*phi)	696.21	192.43	118.93
1/4 comma	5/4	696.58	193.16	117.10
Just 7/4 negative	7/4	696.88	193.76	115.60
Wilson's Meta-Meantone	many values	697.07	194.14	114.65
1/5 comma	15/8	697.65	195.30	111.75
12tET	1200/12	700	200	100
Helmholtz	5/4	701.71	203.42	91.45
Sabat-Garibaldi 1/9 Schisma	6/5	701.74	203.48	91.30
Pythagorean	3/2 and 4/3	701.96	203.91	90.23
7/4 Just	7/4	702.23	204.46	88.85

for more information on Pythagorean tuning and the anomalies and lemmas of this system and

Just Intonation and other historical integer frequency ratio tunings

Many equal temperaments and circular tunings could also be considered meantones, yet after a whole number of steps of fourths or fifths, they arrive at the same position at which they began.

For independent mathematical and graphic analyses of these tuning systems

Go to umich.edu for mathematical comparison of LucyTuning to 1/4 comma meantone and 12tET

Link to Tantrum (Mac program for temperament and tuning analysis and comparison)
More comparisons of LucyTuning to other meantone and equal temperaments.

Meantones are usually considered to be neutral (i.e. 700 cents for the fifth in 12tET); negative (less than 700 cents), or positive (more than 700 cents).

All meantones can be notated in conventional music notation. In negative meantones (like LucyTuning) the sharp of a note will be closer to the starting note than to the Large interval above; whereas in positive meantones it will be nearer to the Larger interval above. i.e in negative meantones L< 2s; in positive meantone L> 2s and in 12tET L=2s.

Return to Chapter One of Pitch, Pi, ...

Get Microsoft Works 4.0 adjustable spreadsheet of fretting positions, and cent + ratio values for any meantone and most xtET tunings. (worksfret.zip)
Information on tuning limitations of synths for playing meantone tunings, and nanotemperaments

Return to LucyTuning homepage

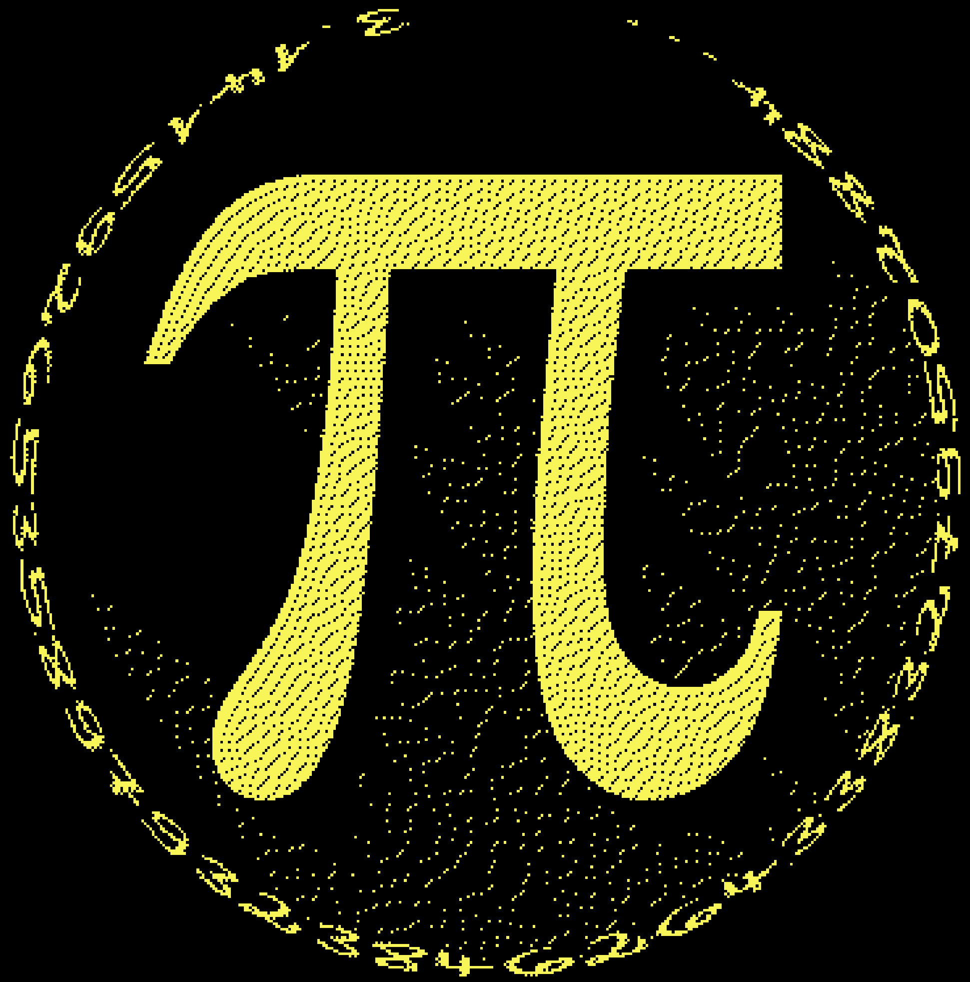
thoughts on:

π & φ

Pi and Phi

π Π

Pi is the ratio of the diameter to the circumference of a circle. It is usually represented by the Greek letter *pi*. There are many ways to derive the value and the calculation of ***pi*** has been refined during history from a simple 22/7 ratio to computer calculations running to many millions of digits. ***Pi*** is both irrational, (its infinitely long string of decimal places never repeats), and transcendental, (it is not a solution of any polynomial equation with whole number coefficients), occurs naturally and is used in all known cultures.

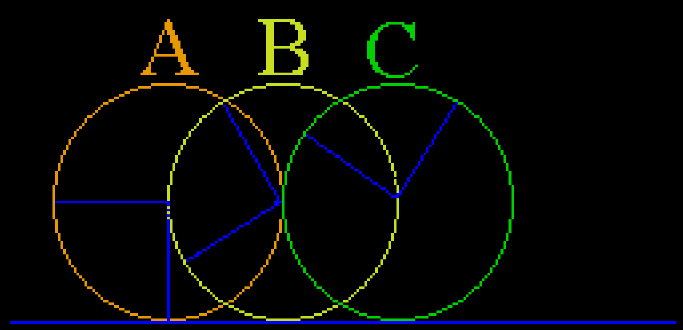


3.14159 26535 89793 23846 26433 83279 50288 41971 etc..

There are many ways to calculate this unique number.
For example

* By continuing the series 1 - 1/3 + 1/5 - 1/7 + 1/9 and multiplying the result by four gives pi.
* Adding the reciprocals of the squares (i.e. 1/1 + 1/4 + 1/9 + 1/9 + 1/16 + 1/25 ....) gives (pi^2)/6.

ϕ Φ



A circle rotated along the blue line from position A to position B turns thru $180/\pi^{0}$ degrees (i.e. one radian).
From A to C through two radians = $360/\pi^{0}$.

Phi is also known as the *Golden* or *Divine Ratio,* and has been used in many models for patterns found in nature and artistic concepts. Mathematically, it is half of the square root of five, plus one....

$$(5^{(1/2)}+1)/2 = 1.618034.....$$

Phi may be derived from the Fibonacci series, as it is the ratio of increase from the series:

1,2,3,5,8,13,21,34,55,89,144............

Phi is the only number which is one greater that its reciprocal.

$$phi = 1 + (1/phi)$$

which is 1.618...... = 1 + (1/1.618.....)

π & φ

Pi and *Phi* are connected mathematically, for *pi* may be calculated from the Fibonacci sequence

The Fibonacci sequence is defined by the formula $u(0) = 0$, $u(1) = 1$
$u(n+1) = u(n) + u(n-1)$
So $u(2) = 1$, $u(3) = 2$, $u(4) = 5$, $u(5) = 8$, $u(6) = 13$, $u(7) = 21$, $u(8) = 34$.........

This connects to pi in the following manner:

Calculate all the Fibonacci numbers from $u(1)$ up to $u(m)$.
Multiply all the results together and call this $P(m)$.
Find the Least Common Multiple $LCM(m)$ from the numbers $u(1)$ through $u(m)$.
(The Least Common Multiple of two or more numbers is the smallest number that can be exactly divided by all the numbers)
Calculate the square root of $6\log P(m)/\log LCM(m)$ and call it $Z(m)$.
As values of *m* increases the value of $Z(m)$ becomes closer to ***pi.***

For *m=7 P(m)* is 1*1*2*3*5*8*13*21 = 32760
LCM(m)=10920 (i.e. 3*5*7*2*2*2*13)
6(log32760/log10920) = 6*4.515/4.038 = 6.709.* Square root is 2.590 = *Z(m)*

For *m=8 P(m)* is 1*1*2*3*5*8*13*21*34= 1113840
LCM(m)=185640
6(log1113840/log185640) = 6*6.0468/5.2687= 6.886.* Square root is 2.6241 = *Z(m)*

This serves to show the method for low numbers. As the values of *m* increase eventually *Z(m)* reaches ***pi***.

More info. on Pi (The Joy of Pi - website)
Pi - The movie [out July 98])

Back to LucyTuning homepage

LucyScaleDevelopments present:

LucyTuned Guitars

The first practical LucyTuned guitar was the Mk V, which was first made in 1986. It has twenty-five frets per octave.

The story of its design and construction can be found in [chapter one of *Pitch, Pi, and Other Musical Paradoxes*](). This is the design used by Arc-Angel and there are now a few hundred copies which have been made in all parts of the world. Although the design was patented, permission has been given for their non-commercial production for personal use. The design details have been evolving, and it is now proposed that new users start with the nineteen fret per octave model, and with experience add twelve further frets to make it thirty-one frets per octave.

Playing LucyTuned guitars

Diagram of LucyTuned and 12tET frettings

This design is intended to make the evolution from 12tET to LucyTuning as easy as possible for experienced musicians and new players. Although all frets, except the octave, are at different positions; the dots or marks found on the neck and fretboard of a conventional guitar are found at directly comparable positions on LucyTuned guitars. The familiar "landmarks" usually found at the 3, 5, 7, 9, and 12th frets on 12tET guitars, are placed at the bIII, IV, V, VI, and VIII positions. (i.e. C, D, E, and F# for the A (5th) string. This enables new users to use familiar open tunings and immediately navigate around the fretboard using familiar fingering.

When playing with more than 19 frets per octave you will use pairs of close frets. Placing your fingers **below** (towards the nut from) **the pair** will sound the **flatter** note. Playing **on the pair** will sound the **sharper** of the two notes.

You will notice that all the **sharper** of the notes sounded from the pairs will be in **sharp keys**; the **lower of the pair** being in **flat keys**.

Tuning LucyTuned guitars

Any tuning of the open strings may be used: conventional (EADGBE), slack key, alternative etc. yet each string will need to be referenced to A4 = 440 Hz. and other notes fine tuned (+/- a few cents). This may be done by matching to frets on adjacent strings, using "harmonics", or an electronic tuner. The tuning needs to be very precise, yet when you have got correct, it will be very apparent, for chords you play will sound very "in tune".

Using conventional tuning the changes are:

Open String	. . . **6** **.** **E**	. . **5** **.** **A**	. **4** **.** **D**	**3** **.** **G**	. . . . . **2** **.** **B**	. . . **1** **.** **E**
Change (cents)	4.5 cents **flat** (**b**)	**same** as 12 tET **110 Hz.**	**4.5** cents **sharp** (#)	**9.0** cents **sharp** (#)	**9.0** cents flat (**b**)	**4.5** cents flat (**b**)

Getting *your own* LucyTuned guitar

There are a number of options.

New Neck and fretboard

New necks can be manufactured for most solid guitars with any specified fretting by: John Carruthers, 346, Sunset Ave, Venice, California 90291. Phone 1 (213) 392-3910 contact Jim Hetal.

Magnetic Fretboard Kits

Mark Rankin, (last seen in Phoenix, AZ.), mail at: Franklin City, Greensbackville, VA 23356, phone contact numbers 1 (714) 688-9894 and 1 (415) 658-1889 can provide a kit for interchangeable fretboards, which are held in position by a magnetic laminate.

DIY: You can produce your own, using magnetic laminate available from: Magna Visual Inc. 9400, Watson Road, St. Louis, Missouri 63126-1598: Voice 1 (314) 843-9000 or Fax 1 (314) 843-0000. ($15 for two sheets .045" x 12" x 24"). You will need to remove all frets and plane or sand down your fretboard to glue on a thin metal sheet (I have used thin galvanised roofing material), then cut the laminate to size, and attach the frets to the laminate.

You can then use the guitar fretless, or with a variety of interchangeable fretting systems.

Refretting

You can get an existing guitar refretted by any competent luthier. I use and recommend: Colin Noden at

Andy's Guitar Shop, Denmark St. London W1. He is very experienced and usually busy, yet does an excellent job. Most luthiers will charge a couple of hundred dollars for the work, and will need the exact fret positions which depend upon your nut to bridge distance.
In the US, you might also try Glen Peterson.
If your instrument has other than 650 mm from nut to bridge, you will need to pro-rata the distances, or EMail to Charles Lucy from here. (lucy@ilhawaii.net) with your nut to bridge distance (in inches or millimeters) to get the AmigaBasic or spreadsheet program or a file of the output.

DIY: Doing it yourself is the least expensive route. Remove all the frets. Fill the holes with Plastic Wood. Allow to dry overnight. Sand the board and stick masking tape over it so that you can mark out the fret positions. Draw a straight line from the centre of the nut to the centre of the bridge as a reference for fret alignment, and mark each fret position. Cut fret grooves; remove the masking tape, insert the frets; secure them; trim; file; dress; set up guitar and enjoy playing your LucyTuned guitar.

I suggest 19 frets per octave initially, so that you can add more frets later as you gain playing experience. Use mandolin fretwire for the second octave, so that there will be space for the extra frets later.

Fret positions for LucyTuned 19 & 31 frets per octave instruments.

Intervals	Ratio	Cents
Large (L)	1.116633	190.9858
small (s)	1.073344	122.5354

Distance from Nut to Bridge = **650** (millimetres)

(for other other nut to bridge distances, values can be pro-rated)

Diagram of LucyTuned and12tET frettings

Note Name Guitar Fifth String (* = marks)	Scale Position	Distance Nut to fret First Octave	Fret No. of **19** for **First octave** (0-19)	Fret No. of **31** for **First octave** (0-31)	Large (L) and small (s) Intervals from nut. Add (5L+2s) for second octave	Distance Nut to fret Second Octave	Fret No. of **19** for **Second octave** (19-38)	Fret No. of **31** for **Second octave** (31-62)
A	I	-	Nut	Nut	Zero & 5L+2s	325.0	19	31
Bbb	bbII	019.9923	-	1	2s-L	334.9962	-	32
A#	#I	025.1987	1	2	L-s	337.5994	20	33
Bb	bII	044.4160	2	3	s	347.2080	21	34
Ax	xI	049.4295	-	4	2L-2s	349.7103	-	35
B	II	067.8928	3	5	L	358.9464	22	36
Cb	bbIII	085.7970	-	6	2s	367.8985	-	37
B#	#II	090.4595	4	7	2L-s	370.2298	23	38

*C	bIII*	107.6696	*5	8*	L+s	378.8348	*24	39*
Dbb	bbIV	124.3503	-	9	3s	387.1751	-	40
C#	III	128.6942	6	10	2L	389.3471	25	41
Db	bIV	144.7283	7	11	L+2s	397.3641	26	42
Cx	#III	148.9038	-	12	3L-s	399.4519	-	43
*D	IV*	164.3163	*8	13*	2L+s	407.1581	*27	44*
Ebb	bbV	179.2546	-	14	L+3s	414.6273	-	45
D#	#IV	183.1449	9	15	3L	416.5724	28	46
Eb	bV	197.5042	10	16	2L+2s	423.7521	29	47
Dx	xIV	201.2436	-	17	4L-s	425.6218	-	48
*E	V*	215.9462	*11	18*	3L+s	432.5231	*30	49*
Fb	bbVI	228.4242	-	19	2L+3s	439.2121	-	50
E#	#V	231.9081	12	20	4L	440.9541	31	51
F	bVI	244.7676	13	21	3L+2s	447.3838	32	52
Ex	xV	248.1164	-	22	5L-s	449.0582	-	53
*F#	VI*	260.4773	*14	23*	4L+s	455.2387	*33	54*
Gb	bbVII	272.4580	15	24	3L+3s	461.2290	34	55
Fx	#VI	275.5781	-	25	5L	462.7890	-	56
G	bVII	287.0943	16	26	4L+2s	468.5472	35	57
Abb	bbVIII	298.2564	-	27	3L+4s	474.1282	-	58
G#	VII	301.1632	17	28	5L+s	475.5816	36	59
Ab	bVIII	311.8925	18	29	4L+3s	480.9462	37	60
Gx	#VII	314.6866	-	30	6L	482.3433	-	61
**A	VIII**	325.0000	19**	31**	5L+2s	487.5000	38**	62**

Back to LucyTuning homepage

Using MIDI Pitchbend and MIDI Tuning Dump to LucyTune your music.

Pitchbend

It is possible to LucyTuned your sequences to the nearest 64th of a semitone using MIDI pitchbend.
Check that your MIDI equipment can recognise and respond to pitchbend data
The current pitchbend range operating on your equipment may be adjustable (see the operating manual), if this is not clear or you wish to test the current range test it as follows:

1. Make a simple MIDI sequence of two adjacent notes (one semitone apart) Eg. G#4 and A4. Play the sequence and you should hear two notes a semitone apart.

2. Now add a pitchbend value to the lower of the two notes, at a value of +4096 and a value of 0 to the higher note. Listen to the two notes again. If the notes are now the same pitch your pitchbend range is set at 4096 units per semitone.

Experiment by changing the values until you get it right, and discover how many pitchbend units are required to bend your equipment by one semitone. This will tell you which column (*d, e, or f*) of the table below to use.

Pitchbend usually effects only the assigned channel, and all notes played on that channel will be "bent" until another pitchbend command is received. Therefore to LucyTune your sequence you may have to move notes to other channels, so that there is only ever one note per channel at any time.

(Remember to reset all the A's to zero)

3. Each note will need to be pitchbent by the appropriate amount. This is a tedious procedure, yet the results can be quite satisfying. You can make the conversion easier by using copy and paste in your sequencer's edit program.

Using the table below and the appropriate pitchbend ranges (columns *d, e, or f)*you can set the pitchbend of each note (column **a**) which you wish to use by adjusting the value for the MIDI note named in column *b.*

The cent values are also included for users of Ensoniq, Korg M series, and other cent programmable equipment (column c*).*

Lucy **Note** Name **a**	**Bend** 12tET Note *b*	**cents** from A4=440Hz c	**Bend** +/- 64ths *d*	**Bend** 2048 ths *e*	**Bend** 4096 ths *f*	**Ratio from A=1.0000** *g*	**Dump** 64ths (Proteus) **h**	**Dump** xx (hex) semitone **i**	**Dump** yy (hex) MSB **j**	**Dump** zz (hex) LSB **k**
C	C	0313.521	+9	+277	+554	1.198531	0009	00	11	27
Dbb	C	0367.605	+43	+1385	+2769	1.236564	0043	00	56	45
C#	C#	0381.972	-12	-369	-738	1.246869	0052	00	68	75
Db	C#	0436.056	+23	+738	+1477	1.286437	0087	01	2E	14

Cx	D	0450.424	-32	-1015	-2031	1.297156	0096	01	40	45
D	D	0504.507	+3	+92	+185	1.338320	0131	02	05	62
Ebb	D	0558.591	+37	+1200	+2400	1.380789	0166	02	4B	00
D#	D#	0572.958	-17	-554	-1108	1.392295	0175	02	5D	31
Eb	D#	0627.042	+17	+554	+1108	1.436478	0209	03	22	4F
Dx	E	0641.410	-37	-1200	-2400	1.448445	0218	03	33	00
E	E	0695.493	-3	-92	-185	1.494412	0253	03	7A	1E
Fb	E	0749.577	+32	+1015	+2031	1.541834	0288	04	3F	3B
E#	F	0763.944	-23	-738	-1477	1.554682	0297	04	51	6C
F	F	0818.028	+12	+369	+738	1.604018	0332	05	17	0A
Gbb	F	0872.112	+46	+1477	+2954	1.654918	0366	05	5C	27
F#	F#	0886.479	-9	-277	-554	1.668709	0375	05	6E	59
Gb	F#	0940.563	+26	+831	+1661	1.721663	0410	06	33	76
Fx	G	0954.931	-29	-923	-1846	1.736018	0419	06	46	28
G	G	1009.014	+6	+185	+369	1.791099	0454	07	0B	45
Abb	G	1063.098	+40	+1292	+2584	1.847936	0488	07	50	62
G#	G#	1077.465	-14	-462	-923	1.863336	0498	07	63	14
Ab	G#	1131.549	+20	+646	+1292	1.922466	0532	08	28	31
Gx	A	1145.917	-35	-1108	-2215	1.938491	0542	08	3A	63
A	A	00000.000	0	0	0	1.000000	0576	09	00	00
Bbb	A	0054.083	+35	+1108	+2215	1.031734	0611	09	45	1D
A#	A#	0068.451	-20	-646	-1292	1.040331	0620	09	57	4F
Bb	A#	0122.535	+14	+462	+923	1.073344	0654	0A	1C	6C
Ax	B	0136.903	-40	-1292	-2584	1.082291	0664	0A	2F	1E
B	B	0190.986	-6	-185	-369	1.116633	0698	0A	74	3B
Cb	B	0245.070	+29	+923	+1846	1.152068	0733	0B	39	58
B#	C	0259.438	-26	-831	-1661	1.161667	0742	0B	4C	0A

Programs to microtune your MIDI files using pitchbend

Oct 1999 A recent trio of PC programs called Midi Tempering Utilities by Fred Nachbaur (free midi pitchbend software) can be used to midi microtune existing midi files.

To LucyTune your files with these programs use the ratios from A = 1.000000 in the *g* column above

MIDI Tuning Dump

In 1992, a new MIDI tuning dump standard was introduced. This is intended to transmit tuning data to a resolution of 16,384 units per semitone, (196,608 per octave). As many of the psychological effects of LucyTuning depend upon subsonic beating: the more accurate the tuning; the greater the effect. Unfortunately I have yet to find a manufacturer who has fully implemented this standard, although it should eventually happen. (Please encourage manufacturers to introduce it in their new products). A number of tuning programs (eg. Tuning Wrench) and some hardware (eg. Proteus 3) already use these values to transfer tuning data.

By using the table above in the Dump columns (**h, i, j, and k**), you can tune to the LucyTuned notes listed in column **a.** The tuning resolution which is played will depend upon your hardware. The 64th of a semitone (Proteus) and xx (hex) values shown in the table are for the lowest octave. For higher octaves add 768 units per octave to the 64th of a semitone (Proteus) column (**h**), or 12 (0C in hex) per octave to the xx column (**i**).

[Frequency data format (all bytes in hex)]

xx semitone = 100 cent units; yy MSB (Most Significant Byte) of fraction (1/128 semitone) = 0.78125 cent units; LSB (Least Significant Byte) of fraction (1/16384 semitone) = 0.0061 cent units

[LucyTuning table of 52 arrangements for microtunable synths and samplers]

[Back to LucyTuning homepage]

Pitch, Pi. and Other Musical Paradoxes (a practical guide to natural microtonality)

by Charles E. H. Lucy

Ever wondered??????? *

* How many notes there should be in an octave?

* Why the black note between G and A has two or more names?

* Why some keys are called sharp whilst others are flat?

* Why the music of some other cultures use different scales and tuning systems?

* Why conventional guitars may sound in tune for some keys, in some positions, but very out of tune in others, and how to refret them?

* Whether the musical circle of fourths and fifths should really be a spiral, a torus, or a cylinder?

* How to microtune the Yamaha DX7 MkII, TX81Z, Korg M1, Synclaviers and many Roland and Ensoniqs and other instruments have microtonal capability? (Includes MIDI tuning dump and pitchbend data)

* How pitch and colour are connected?

* How music is related to quantum physics, longitude, and cosmology?

* Why musical tuning was of paramount importance to Chinese emperors?

* Whether what the music colleges taught about harmony is based on some fundamentally false premises?

* Why some foreign music sounds out of tune?

* How musical scales are related to the only irrational and transcendental number which occurs in all cultures naturally (*pi*)?

Read on.........

Synopsis

THE PROBLEM

There are many mythological references to the music of the spheres, and countless learned attempts to construct a unifying theory to explain the relationship between music, physics, astronomy, and mathematics. No single theory has become generally accepted, yet the search continues to find patterns in music which, in some mysterious way, reflect patterns fundamental to the nature of the universe. Recent discoveries in quantum physics and mathematics suggest that this link is not so tenuous as it has seemed. The musical aspects of this puzzle are particularly paradoxical; for although contemporary Western society divides the musical octave into twelve equal parts (semitones), this is merely a convenient compromise to represent an underlying organisation of frequencies which are only now fully understood. Thousands of ways had been devised to split the octave into discrete intervals. Other temperaments persist in the diverse musical traditions found today as ethnic music or revivals of older scales and tunings.

THE RESEARCH

Many of the alternative temperaments are well documented. One system, proposed by the British horologist **John Harrison** (*1693-1776*), is unique in that he uses the ratio of the diameter to the circumference of the circle (i.e. *pi = 3.14159 26535 etc*.) as the basis of his 'natural' scale.

THE SOLUTION

This tuning system uses a Large interval and a small interval. By selecting permutations and combinations of these two intervals, an infinite number of notes may be computed, which can represent any possible scale and hence produce a universal musical notation, and harmonic mapping.

THE IMPLICATIONS

Using this tuning system, fretted instruments have been built with nineteen, twenty-five and thirty-one frets per octave, and synthesisers programmed, to play it. Scales and harmonic structures have been analysed to create a '*Musical Esperanto*', on which any instrument may be build or adapted to play in any key or modality, using conventional Western musical notation. Using LucyTuning, the circle of fourths and fifths which equal temperament uses as its harmonic basis, have been found to be a spiral of fourths and fifths expanding octaves in fourths for flat keys, and contracting in fifths for sharp keys. The use of this scale, opens musical possibilities for the re-interpretation of existing music, and unlimited potential for new composition. Adjacent sharps and flats which are assumed to be of the same frequency in conventional harmony, may now be treated as separate pitches. This increases the tonal vocabulary as the extra altered notes may also modulate into double, triple, or more sharps or flats, giving greater pitch choice and precision, which matches natural harmonics.

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latest hardcopy available at $(US)330,

Visa/Mastercard/American Express Accepted. Price includes p. & p. plus on-line EMail/Internet support service for registered users from from lucy@hour.com.

There is also a lucyscaledevel conference on **cix.compulink.co.uk**.

|LucyTuning homepage|

LucyScaleDevelopments presents extracts from:

Pitch, Pi, and Other Musical Paradoxes (a practical guide to natural microtonality)

Chapter Two

FOURTHS AND FIFTHS: CIRCLES, SPIRALS, OR CYLINDERS?

Twelve tone Equal Temperament (12tET)

In twelve note equal temperament, as the octave is divided into twelve equal intervals, it is possible to construct a circle of twelve notes in intervals of fourths of (500 cents) in a clockwise direction and of fifths (700 cents) in the opposite direction. Moving twelve steps in either direction will arrive back at the starting note in the next octave.

(see clockface diagram of 12tET)

Spiral of fourths and fifths.

This principle of cumulative fifths is also used to arrive at the pitches of the Pythagorean tuning, which like many of the fractional scales, uses the ratio of 3:2 for the fifth = ratio of 1.5.. The Pythagorean tuning having completed this circle arrives at a ratio of 531441:262144 = 2.0272865 and hence a spiral (see Chapter One of Pitch, Pi,....)

LucyTuning spiral

In LucyTuning, a sharp and its adjacent flat are not assumed to be the same pitch, instead of a circle of fourths and fifths; there is an expanding spiral of fourths (504.51 cents) and a contracting spiral of fifths (695.49 cents).

Twelve LucyTuned fourth intervals; 1.33832012 = 33.015632 instead of (32.00000 for 5 octaves) or 1.333333312 = 31.569292 (12* 504.51) = 6054.12 cents

compared to 1200 ^ 5 = 6000 cents i.e. 6054.12 - 6000 = 54.12 cents sharper, for each fourths revolution of the spiral. and for twelve fifth intervals (1.49441112 = 124.06254 instead of (128.0000 for seven octaves) or 1.5^12 = 129.74634 (12* 695.49) = 8345.88 cents compared to 1200 ^ 7 = 8400 cents i.e. 8400 - 8345.88 = 54.12 cents flatter, for each fifths revolution of the spiral. [This is the bbIInd interval 2s-L].

See diagram of spiral

To find the recipe for making a physical model

As this scale has an infinite number of intervals, the sharpened notes become closer to the adjacent flattened notes as the number of intervals per octave increases. By increasing the number of notes per octave, eventually adjacent pitches become too close for the human ear to distinguish between them. Any interval may therefore be described in musical terms of single or multiple sharps or flats, as shown below.

The significance of the columns is as follows:

The leftmost five columns are for the cycle of fifths.

Name. This is the name of the note starting from A. and follows the sequence A E B F# C# G# D# after which the sequence repeats with one extra sharp A# E# B# F## C## G## D## and continues the next step with A## etc.

Position in scale. This column shows the position in the A Major scale. Remember A Major has three sharps. The scale positions are expressed in Roman numerals. A=I B=II C#=III D=IV E=V F#=VI G#=VII. As with the note names the pattern is again repeated after seven steps and for the fifths is I V II VI III VII #IV followed by I# V# II# etc.

Cents from A. This column shows the interval upwards from A to the nearest named note, expressed in cents (1200 cents = one octave).

Large and small intervals (L&s). This column shows the number of Large and small intervals from which this interval is also derived. The values are always multiple addition and subtraction of whole Large and small intervals. The sequence of the pattern in this column (for fifths) is continued additions of 3L+s. So that for the second step the value is (3L+s)*2 = (6L+2s), but since this now takes us above the first octave and into the second it has been reduced by (5L+2s) to give a value of less than 1200 cents, and therefore (6L+2s)-(5L+2s)= L, which is less than one octave above our starting point. To find the value for any step of fifths or {fourths} multiply the step number by (3L+s) or {2L+s} and subtract the nearest number of whole octaves (5L+2s) below. The result is your remainder and the value for this step in the first octave.

Hertz This is the frequency of the named note in the octave between A2=110 Hz. and A3=220 Hz.

Step number. Surprisingly, this is exactly what it says; the number of steps in fifths or fourths from the starting point of A2=110Hz, 0 cents, as the tonic (I). The rightmost five columns are the equivalent columns for fourths and are the mirror image of the columns explained above. The fourth interval is (2L+s), and the note name, and scale position sequences are the exact reverse of those for the fifths. You will notice that for each step the fourths columns added to the fifths columns exactly equals one octave.

Table of first 43 Note Names, Hertz and Cents for cumulative fifths and fourths (A=110 Hz.)

This table shows the result of cumulative fourth and cumulative fifth intervals. Large (L) and small (s)intervals are shown from A at 110 Hertz. The cent values are in relation to A and the frequencies assume A=110 Hz.

Note name	Position	cents from A	L & s from A	Hertz	Steps	Note name	Position	cents from A	L & s from A	Hertz
A	I	-------	-----	110.000	00	A	I	-------	-----	110.000
E	V	0695.493	**3L+s**	164.385	01	D	IV	0504.507	**2L+s**	147.215
B	II	0190.986	L	122.830	02	G	bVII	1009.014	4L+s	197.021
F#	VI	0886.479	4L+s	183.558	03	C	bIII	0313.521	L+s	131.838
C#	III	0381.972	2L	137.156	04	F	bVI	0818.028	3L+2s	176.442

G#	VII	1077.465	5L+s	204.967	05	Bb	bII	0122.535	s	118.068
D#	#IV	0572.958	3L	153.153	06	Eb	bV	0627.042	2L+2s	158.013
A#	#I	0068.451	L-s	114.436	07	Ab	bVIII	1131.549	4L+3s	211.471
E#	#V	0763.944	4L	171.015	08	Db	bIV	0436.056	L+2s	141.508
B#	#II	0259.438	2L-s	127.784	09	Gb	bbVII	0940.563	3L+3s	189.383
Fx	#VI	0954.931	5L	190.961	10	Cb	bbIII	0245.070	2s	126.727
Cx	#III	0450.424	3L-s	142.687	11	Fb	bbVI	0749.577	2L+3s	169.602
Gx	#VII	1145.917	6L	213.234	12	Bbb	bbII	0054.084	2s-L	113.491
Dx	xIV	0641.410	4L-s	159.329	13	Ebb	bbV	0558.591	L+3s	151.887
Ax	xI	0136.903	2L-2s	119.052	14	Abb	bbVIII	1063.098	3L+4s	203.273
Ex	xV	0832.396	5L-s	177.912	15	Dbb	bbIV	0367.605	3s	136.022
Bx	xII	0327.889	3L-2s	132.937	16	Gbb	3bVII	0872.112	2L+4s	182.041
F3#	xVI	1023.382	6L-s	198.663	17	Cbb	3bIII	0176.619	3s-L	121.815
C3#	xIII	0518.875	4L-2s	148.442	18	Fbb	3bVI	0681.126	L+4s	163.027
G3#	xVII	0014.368	2L-3s	110.917	19	B3b	3bII	1185.633	3L+5s	218.182
D3#	3#IV	0709.861	5L-2s	165.755	20	E3b	3bV	0490.140	4s	145.999
A3#	3#I	0205.354	3L-3s	123.853	21	A3b	3bVIII	0994.647	2L+5s	195.393

This progression continues infinitely. Pi is both an irrational and a transcendental number. Each further step will produce two new and unique values and hence more intervals and positions. Notice that the patterns of note names and note positions repeat themselves every seven steps adding an extra # through the fifths and b through the fourths.

For each step the cents, intervals, and note positions from the fifths added to the corresponding cents, intervals and note names from the fourths result in an exact octave.

Eg. after nine steps 763.94 + 436.06 = 1200.00 cents; #V + bIV = VIII and 4L + (L+2s) = 5L +2s.

|Graph of this pattern and comparison to integer frequency ratios| |Chapter One| |Chapter Three|

Return to LucyTuning homepage

You may have noticed that the intervals which are closest on the spiral of fourths and fifths tend to sound more consonant. To download more technical information visit our homepage extract from CHAPTER THREE of

Pitch, Pi, And Other Musical Paradoxes

SCALES ON THE LUCY SCALE

12tET

Equal Temperament twelve notes per octave (12tET) assumes that there is only one interval on which all musical scales are based. This interval is the semitone (100 cents). Twelve equal semitones make one octave. Two semitones make one whole tone.

LucyTuning

The naturals arranged in order of fifths gives: **F-C-G-D-A-E-B** *1 2 3 4 5 6 7* This is a chain of seven notes, an expanse of six steps. Hence the first digit in the code is 6/ None of these notes are missing, so the second digit is 6/0. Each scale begins on the notes in positions 1 through 7 as follows:

So for Scales Using All The Naturals.

Notice that each octave consists of 5 Large (L) and 2 small (s) intervals in the circular sequence LLsLLLs with each scale starting from a different position in the sequence.

1	2	3	4	5	6	7	8	Scale Coding	Greek Name & Interval Pattern	Indian Name
C	D	E	F	G	A	B	C	6/0/2	Ionian LLsLLLs	Bilaval
D	E	F	G	A	B	C	D	6/0/4	Dorian LsLLLsL	Kafi
E	F	G	A	B	C	D	E	6/06	Phrygian sLLLsLL	Bhairavi
F	G	A	B	C	D	E	F	6/0/1	Lydian LLLsLLs	Kalyan
G	A	B	C	D	E	F	G	6/0/3	Mixolydian LLsLLsL	Khamaj
A	B	C	D	E	F	G	A	6/0/5	Aeolian LsLLsLL	Asavari
B	C	D	E	F	G	A	B	6/0/7	Locrian sLLsLLL	-

Table of more than 60 scales For more information on LucyTuned ScaleCoding
conventional keyboard layout for modulation of pattern (.gif)

Pitch Pi,....**|Chapter One| |Chapter Two|**

|LucyTuning homepage|

Some of the scales and intervals available using LucyTuning

Clicking in the rectangle area for each scale will trigger a LucyTuned **midi** file for you to hear

12tET.mid

More than sixty of the thousands of scales you can play using LucyTuning ® & LucyTuning technology

	Scale Names / Mode Code / Intervals	1	2	3	4	5	6	7
1	Ionian / *Bilaval* 6/0/2 (White notes on piano ascending from C up to B)	C L	D L	E s	F L	G L	A L	B s
2	Dorian (Western Name) 6/0/4 *Kafi (Indian Name)* (Naturals D-C)	C L	D s	Eb L	F L	G L	A s	Bb L
3	Phrygian / *Bhairavi* 6/0/6(Naturals E-D)	C s	Db L	Eb L	F L	G s	Ab L	Bb L
4	Lydian / *Kalyan* 6/0/1 (Naturals F-E)	C L	D L	E L	F# s	G L	A L	B s
5	Mixolydian / *Khamaj* 6/0/3 (Naturals G-F)	C L	D L	E s	F L	G L	A s	Bb L
6	Aeolian / *Asavari* 6/0/5 (Naturals A-G)	C L	D s	Eb L	F L	G s	Ab L	Bb L
7	Locrian 6/0/7 (Naturals B-A)	C s	Db L	Eb L	F s	Gb L	Ab L	Bb L
8	Melodic Minor 8/28/4	C L	D s	Eb L	F L	G L	A L	B s
8d	*(descends as Aeolian)*					s	Ab L	Bb L
9	Neapolitan 10/2481 0/6	C s	Db L	Eb L	F L	G L	A L	B s
10	Neapolitan Minor 10/48910/6	C s	Db L	Eb L	F L	G s	Ab (2L-s)	B s
11	Harmonic Minor 9/389/5	C L	D s	Eb L	F L	G s	Ab (2L-s)	B s
12	Hungarian Minor 10/3489/5	C L	D s	Eb (2L-s)	F# s	G s	Ab (2L-s)	B s
13	Romanian 9/389/4	C L	D s	Eb (2L-s)	F# s	G L	A s	Bb L
14	Hungarian Folk or Byzantine 10/3489/6	C s	Db (2L-s)	E s	F L	G s	Ab (2L-s)	B s
	Indian Name scales							
15	Bhairav 9/238/6	C s	Db (2L-s)	E s	F L	G L	A s	Bb L
16	Marva / Marvi 11/23458/6	C s	Db (2L-s)	E L	F# s	G L	A L	B s
17	Purvi bVI 11/34589/6	C s	Db (2L-s)	E L	F# s	G s	Ab (2L-s)	B s
18	Purvi bVII 11/235811/6	C s	Db (2L-s)	E L	F# s	G L	A s	Bb L
19	Todi bVI 11/458910/6	C s	Db L	Eb (2L-s)	F# s	G s	Ab (2L-s)	B s
20	Todi bVII 11/2581011/6	C s	Db L	Eb (2L-s)	F# s	G L	A s	Bb L
21	Persian 11/458910/7	C s	Db (2L-s)	E s	F s	Gb L	Ab (2L-s)	B s
22	Spanish Folk & Jewish Major 9/389/6	C s	Db (2L-s)	E s	F L	G s	Ab L	Bb L
23	Enigmatic / Verdi 15/2345789131 5/6	C s	Db (2L-s)	E L	F# L	G# L	A# s	B s
24	Stravinski example 10/24810/5	C L	D L	E L	F# s	G s	Ab L	Bb L
25	Whole Tone alternate 12/2468101 2/1	C L	D L	E L	F# L	G# L	A# L	B# (2s-L)
26	Hindi bVI & bVII 8/28/5	C L	D L	E s	F L	G s	Ab L	Bb L
27	Hindi #IV & #V 8/28/1	C L	D L	E L	F# L	G# s	A L	B s
28	Hindi #IV & bVII or Dominant 7th Lydian 8/28/3	C L	D L	E L	F# s	G L	A s	Bb L
29	Hindi 3 flats & bV 8/28/7	C L	D s	Eb L	F s	Gb L	Ab L	Bb L
30	Hindi 5 flats & bIV 8/28/9	C s	Db L	Eb s	Fb L	Gb L	Ab L	Bb L
31	Hindi bII, bIII & bVII 8/28/6	C s	Db L	Eb L	F L	G L	A s	Bb L
32	Damien Emanuel 10/48910/5	C L	D s	Eb (2L-s)	F# s	G s	Ab L	Bb L
33	Pseudo Turkish 9/389/7	C s	Db L	Eb L	F s	Gb (2L-s)	A s	Bb L

JAPANESE

Mode Code and Classification

x/m1 m2/T. *x* = The expanse. i.e. The number of steps along the chain of fourths and fifths from the flatmost fourth note name to the sharpmost fifth for each scale.
m1 m2 etc. = position of missing notes. *T* = Tonic.
Eg. 9/389/7 Pseudo Turkish (last listing above)

Chain is 9 steps (from Gb to A) –
Notes 3 (Ab), 8 (G), and 9 (D) are

.mid 12tET

SOME OF THE PENTATONIC POSSIBILITIES

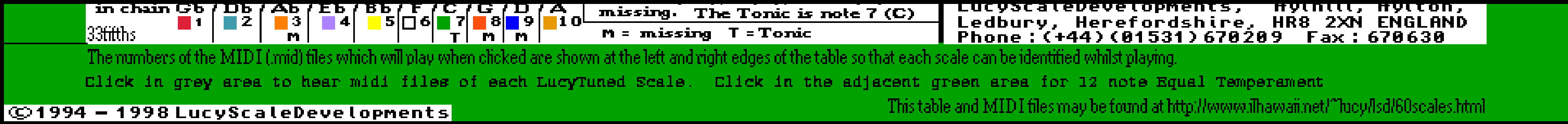
Blues Scales	1	2	3	4	5	
Blues bV 6/23/7	C (L+s)	Eb L	F s	Gb (2L)	Bb L	34
Blues #IV 9/25678/4	C (L+s)	Eb L	F (L-s)	F# (2L+s)	B s	35
Minor Blues 4/0/4	C (L+s)	Eb L	F L	G (L+s)	Bb L	36
Country and Old Chinese 4/0/1	C L	D L	E (L+s)	G L	A (L+s)	37
Scriabin 9/23458/6	C s	Db (2L-s)	E (L+s)	G L	A (L+s)	38
Oriental 5/4/3	C L	D (L+s)	F (2L)	A (L+s)	Bb L	39
Ancient Egyptian & Indian 6/56/3	C (2L)	E s	F L	G (L+s)	Bb L	40
Example of Slendro 11/4thru10/12	C (2s)	Ebb (2L-s)	F (2s)	Abb L	Bbb (2L-s)	41
Japanese Scales						
Ritusen 4/0/2	C L	D (L+s)	F L	G L	A (L+s)	42
Hirajoshi 6/35/4	C L	D s	Eb (2L)	G s	Ab (2L)	43
Kumoi 6/23/4	C L	D s	Eb (2L)	G L	A (L+s)	44
Iwato 6/34/7	C s	Db (2L)	F s	Gb (2L)	Bb L	45
Soft Ascend 6/23/6	C s	Db (2L)	F L	G (L+s)	Bb L	46a
Soft Descend 6/34/6	C s	Db (2L)	F L	G s	Ab (2L)	46d
Hard Ascend 4/0/4	C (L+s)	Eb L	F L	G (L+s)	Bb L	47a
Hard Descend 4/0/2	C L	D (L+s)	F L	G L	A (L+s)	47d

Example of Pelog 22/2346781112 1316 thru21/9: C (2L-2s) · Cx (4s-L) · Fb (2L-2s) · F# s · G s · Ab (2L-s) · B s — 48

Some other 6&8 note scales

1	2	3	4	5	6	7	8	Mode Code & Classification	
C (L+s)	Eb L	F s	Gb (L-s)	G (L+s)	Bb L			7/23/7	49
C (L+s)	Eb L	F (L-s)	F# s	G (L+s)	Bb L			9/678/4	50
C L	D L	E L	F# L	G# L	A# (2s)			10/246810/1	51
C L	D L	E (2s)	Gb L	Ab L	Bb L			10/246810/7	52
C L	D L	E L	F# (L+s)	A s	Bb L			8/248/3	53
C s	Db (2L-s)	E L	F# (L+s)	A s	Bb L			11/2357811/6	54
C L	D s	Eb L	F (L-s)	F# s	G s	Ab L	Bb L	10/8910/5	55
C s	Db L	Eb L	F (L-s)	F# L	G# s	A L	B s	13/24781013/6	56
C s	Db L	Eb (L-s)	E s	F s	Gb L	Ab L	Bb L	10/8910/7	57
C L	D s	Eb L	F L	G s	Ab L	Bb (L-s)	B s	9/89/5	58
C (L-s)	C# s	D L	E L	F# s	G L	A L	B s	7/0/1	59
C L	D L	E s	F (L-s)	F# s	G L	A L	B s	7/0/2	60
C L	D L	E s	F L	G L	A s	Bb (L-s)	B s	7/0/3	61
C L	D s	Eb (L-s)	E s	F L	G L	A s	Bb L	7/0/4	62

Interval to note above is listed under each note name
L = Large interval (II) = 2(1/(2*Pi)) = 1.116633
= 190.9858 ¢
s = small interval (bII) = 122.5354 ¢
(2s-L) = bbII (L-s) = #I
(2s) = bbIII (2L-s) = #II (L+s) = bIII (2L) = III



This diagram is also in the LucyTuned Lullabies (from around the world) booklet.

|Scalemaking| |More diagrams about LucyTuning| |LucyTuning homepage|

LucyScaleDevelopments presents extracts from:

"Pitch, Pi, and Other Musical Paradoxes (a practical guide to natural microtonality)"

by Charles E. H. Lucy copyright 1986-1997

Recipe for a 3D Physical Model display of LucyTuning

Ingredients:

Length of flexible wire (8 to 10 metres) (25 to 30 feet) or 5-8 wire coat hangers.

50 (Elastic) rubber bands.

31 wooden clothes pegs (clothes pins) or 31 tags or sticky labels for pegs.

5 one metre (3 foot) lengths of different colored string or wool..

One cylindrical can.

(For optional coloured note positions) Palette of paints and brushes. Pitch to colour

Tools required:

One protractor.

Directions:

To prepare coat hangers:

Unbend coat hangers. Join ends of straightened wires end to end with tape or weld to provide a 8 to 10 metres continuous length of wire.

Construction from wire: Bend wire around a conveniently sized cylindrical tin (can) to form spiral spring shape of 8-12 inch (200-400mm) diameter.

Position, place or tie 31 elastic bands for markers at intervals of 208.65 degrees around the spiral spring using protractor.

Label thirty-one clothes pegs (pins). One for each note name: Fbb Cbb Gbb Dbb Abb Ebb Bbb Fb Cb Gb Db Ab Eb Bb F C G D A E B F# C# G# D# A# E# B# F## C## G##.

Attach labeled clothes pegs: Suspend the wire spiral with its elastic band markers by a loop bent from the wire after your last mark. Eg. From a lamp or ceiling. Attach the clothes pegs in the above sequence (at the marks, beginning at the end you started your angle measurements from. (the bottom of the spiral), and ascending by one fifth (208.65 degrees) each step.

There are many possible modifications for this basic design, eg. paint markings, color code pegs. blue tack, or chewing gum instead of elastic bands, although the angle and spiral shape should be precise.

Display various scales

1. *Greek Modes and Indian equivalents using only naturals,* attach a length of string to join the pegs: C-D-E-F-G-A-B and back to C.

You will notice that the Large intervals move upwards and one radian around the spiral. The small intervals move downwards and 68.45 degrees around the spiral.

Starting at each of the points and listing them in ascending order will give you the seven Greek modes (Indian names in brackets):

starting from C=Ionian (Indian Bilaval Scale); D=Dorian (Kafi); E=Phrygian (Bhairavi); F=Lydian (Kalyan); G=Mixolydian (Khamaj); A=Aeolian (Asavari; and B=Locrian Mode.

2. *Sharps and flats* Attach second and third strings of different colours in the following sequences:

1) G#-A#_B#-C#-D#-E#-F##-G#

2) Ab-Bb-C-D-Eb-F-G-Ab

You will see that you have generated the same shapes, but higher and lower on the spiral and with a small phase shift between them of (2s-L)=54.084" or 16.2252 degrees

(1) Up into the sharp tonalities [G# Major scale (Ionian) i.e. 8 sharps]

(2) Down into the flat tonalities [Ab Major (Ionian) Scale i.e. 4 flats]

3. *Hindi Scale* Tie strings to connect the following sequence: C-D-E-F-G-Ab-Bb-C

This produces the shape of one of the fundamental Hindi scales. You will notice that it follows the sequence of the Ionian mode up to G but diverges after that as though it was a flat Western scale having three flats. You could consider it as an Ab Major with the Eb sharpened to E in Western terms or as the root of five further modes following the pattern of four adjacent Large intervals and a single Large separated by two separate small intervals. (LLsLsLL).

4. *The Mysterious Middle Eastern Interval.* East of a line North/South from about where the Iron Curtain used to stand, there is a cultural area which uses an interval absent from Western music. This interval is called the sharp second and is the difference between two Large and one small interval ***(2L-s)***. Examples of this interval are found in many Indian, Hungarian, and Romanian scales. You can represent it on your model by joining Ab and B; Db and E; Eb and F#, or any other way which gives the same angles or phase.

I trust these instructions have given you a taste for the geometry of musical harmonics. This is just a beginning. You can build your own new musical scales by connecting the pegs in any way which makes sense to you, and produce new modes, scales, and harmonies. Have fun with it.

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lucy@hour.com Nov, 1997.

Back to LucyTuning home

Pitch, Pi. and Other Musical Paradoxes (a practical guide to natural microtonality)

by Charles E. H. Lucy

Ever wondered??????? *

* How many notes there should be in an octave?

* Why the black note between G and A has two or more names?

* Why some keys are called sharp whilst others are flat?

* Why the music of some other cultures use different scales and tuning systems?

* Why conventional guitars may sound in tune for some keys, in some positions, but very out of tune in others, and how to refret them?

* Whether the musical circle of fourths and fifths should really be a spiral, a torus, or a cylinder?

* How to microtune the Yamaha DX7 MkII, TX81Z, Korg M1, Synclaviers and many Roland and Ensoniqs and other instruments have microtonal capability? (Includes MIDI tuning dump and pitchbend data)

* How pitch and colour are connected?

* How music is related to quantum physics, longitude, and cosmology?

* Why musical tuning was of paramount importance to Chinese emperors?

* Whether what the music colleges taught about harmony is based on some fundamentally false premises?

* Why some foreign music sounds out of tune?

* How musical scales are related to the only irrational and transcendental number which occurs in all cultures naturally (*pi*)?

Read on.........

Synopsis

THE PROBLEM

There are many mythological references to the music of the spheres, and countless learned attempts to construct a unifying theory to explain the relationship between music, physics, astronomy, and mathematics. No single theory has become generally accepted, yet the search continues to find patterns in music which, in some mysterious way, reflect patterns fundamental to the nature of the universe. Recent discoveries in quantum physics and mathematics suggest that this link is not so tenuous as it has seemed. The musical aspects of this puzzle are particularly paradoxical; for although contemporary Western society divides the musical octave into twelve equal parts (semitones), this is merely a convenient compromise to represent an underlying organisation of frequencies which are only now fully understood. Thousands of ways had been devised to split the octave into discrete intervals. Other temperaments persist in the diverse musical traditions found today as ethnic music or revivals of older scales and tunings.

THE RESEARCH

Many of the alternative temperaments are well documented. One system, proposed by the British horologist **John Harrison** (*1693-1776*), is unique in that he uses the ratio of the diameter to the circumference of the circle (i.e. *pi = 3.14159 26535 etc*.) as the basis of his 'natural' scale.

THE SOLUTION

This tuning system uses a Large interval and a small interval. By selecting permutations and combinations of these two intervals, an infinite number of notes may be computed, which can represent any possible scale and hence produce a universal musical notation, and harmonic mapping.

THE IMPLICATIONS

Using this tuning system, fretted instruments have been built with nineteen, twenty-five and thirty-one frets per octave, and synthesisers programmed, to play it. Scales and harmonic structures have been analysed to create a '*Musical Esperanto*', on which any instrument may be build or adapted to play in any key or modality, using conventional Western musical notation. Using LucyTuning, the circle of fourths and fifths which equal temperament uses as its harmonic basis, have been found to be a spiral of fourths and fifths expanding octaves in fourths for flat keys, and contracting in fifths for sharp keys. The use of this scale, opens musical possibilities for the re-interpretation of existing music, and unlimited potential for new composition. Adjacent sharps and flats which are assumed to be of the same frequency in conventional harmony, may now be treated as separate pitches. This increases the tonal vocabulary as the extra altered notes may also modulate into double, triple, or more sharps or flats, giving greater pitch choice and precision, which matches natural harmonics.

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latest hardcopy available at $(US)330,

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|LucyTuning homepage|

LucyScaleDevelopments presents extracts from:

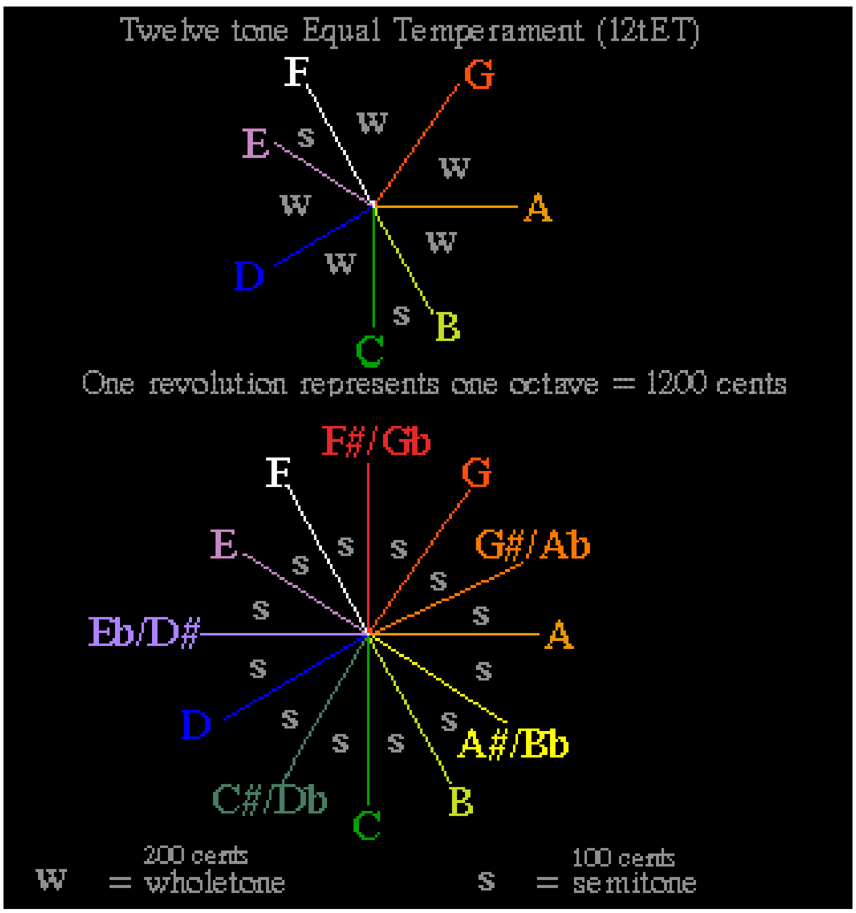
Pitch, Pi, and Other Musical Paradoxes (a practical guide to natural microtonality)

Chapter Two

FOURTHS AND FIFTHS: CIRCLES, SPIRALS, OR CYLINDERS?

Twelve tone Equal Temperament (12tET)

In twelve note equal temperament, as the octave is divided into twelve equal intervals, it is possible to construct a circle of twelve notes in intervals of fourths of (500 cents) in a clockwise direction and of fifths (700 cents) in the opposite direction. Moving twelve steps in either direction will arrive back at the starting note in the next octave.



Spiral of fourths and fifths.

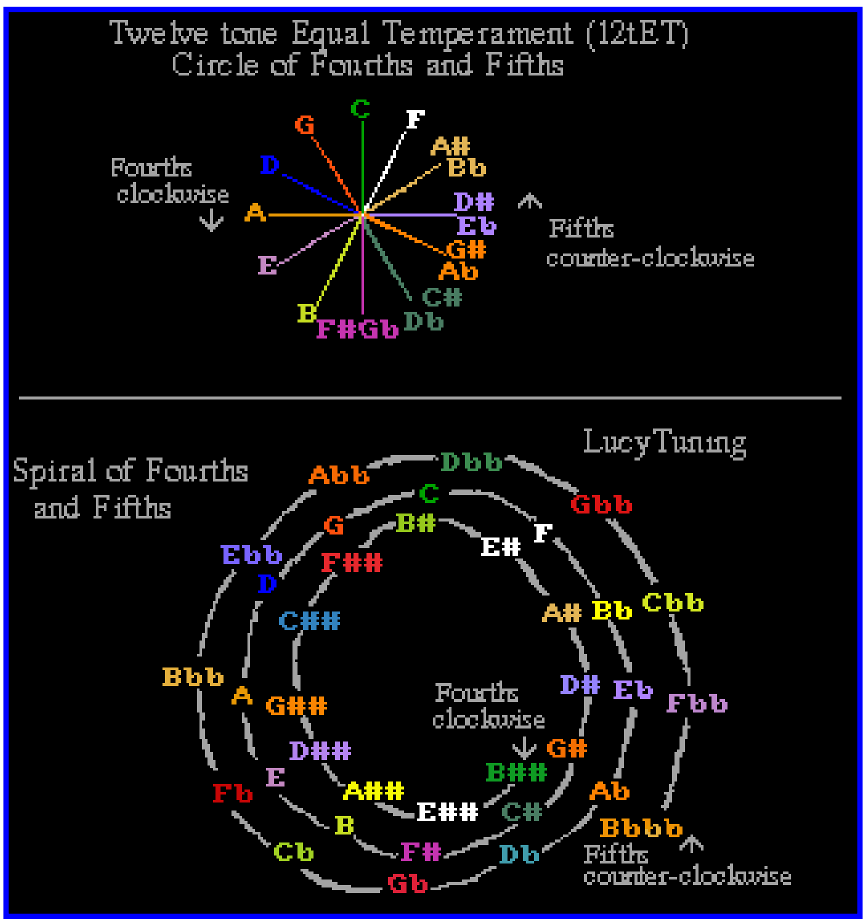
This principle of cumulative fifths is also used to arrive at the pitches of the Pythagorean tuning, which like many of the fractional scales, uses the ratio of 3:2 for the fifth = ratio of 1.5.. The Pythagorean tuning having completed this circle arrives at a ratio of 531441:262144 = 2.0272865 and hence a spiral (see Chapter One of Pitch, Pi,....)

LucyTuning spiral

In LucyTuning, a sharp and its adjacent flat are not assumed to be the same pitch, instead of a circle of fourths and fifths; there is an expanding spiral of fourths (504.51 cents) and a contracting spiral of fifths (695.49 cents).

Twelve LucyTuned fourth intervals; 1.33832012 = 33.015632 instead of (32.00000 for 5 octaves) or 1.333333312 = 31.569292 (12* 504.51) = 6054.12 cents

compared to 1200 ^ 5 = 6000 cents i.e. 6054.12 - 6000 = 54.12 cents sharper, for each fourths revolution of the spiral. and for twelve fifth intervals (1.49441112 = 124.06254 instead of (128.0000 for seven octaves) or 1.5^12 = 129.74634 (12* 695.49) = 8345.88 cents compared to 1200 ^ 7 = 8400 cents i.e. 8400 - 8345.88 = 54.12 cents flatter, for each fifths revolution of the spiral. [This is the bbIInd interval 2s-L].



[To find the recipe for making a physical model](#)

As this scale has an infinite number of intervals, the sharpened notes become closer to the adjacent flattened notes as the number of intervals per octave increases. By increasing the number of notes per octave, eventually adjacent pitches become too close for the human ear to distinguish between them. Any interval may therefore be described in musical terms of single or multiple sharps or flats, as shown below.

The significance of the columns is as follows:

The leftmost five columns are for the cycle of fifths.

Name. This is the name of the note starting from A. and follows the sequence A E B F# C# G# D# after which the sequence repeats with one extra sharp A# E# B# F## C## G## D## and continues the next step with A## etc.

Position in scale. This column shows the position in the A Major scale. Remember A Major has three sharps. The scale positions are expressed in Roman numerals. A=I B=II C#=III D=IV E=V F#=VI G#=VII. As with the note names the pattern is again repeated after seven steps and for the fifths is I V II VI III VII #IV followed by I# V# II# etc.

Cents from A. This column shows the interval upwards from A to the nearest named note, expressed in cents (1200 cents = one octave).

Large and small intervals (L&s). This column shows the number of Large and small intervals from which this interval is also derived. The values are always multiple addition and subtraction of whole Large and small intervals. The sequence of the pattern in this column (for fifths) is continued additions of 3L+s. So that for the second step the value is (3L+s)*2 = (6L+2s), but since this now takes us above the first octave and into the second it has been reduced by (5L+2s) to give a value of less than 1200 cents, and therefore (6L+2s)-(5L+2s)= L, which is less than one octave above our starting point. To find the value for any step of fifths or {fourths} multiply the step number by (3L+s) or {2L+s} and subtract the nearest number of whole octaves (5L+2s) below. The result is your remainder and the value for this step in the first octave.

Hertz This is the frequency of the named note in the octave between A2=110 Hz. and A3=220 Hz.

Step number. Surprisingly, this is exactly what it says; the number of steps in fifths or fourths from the starting point of A2=110Hz, 0 cents, as the tonic (I). The rightmost five columns are the equivalent columns for fourths and are the mirror image of the columns explained above. The fourth interval is (2L+s), and the note name, and scale position sequences are the exact reverse of those for the fifths. You will notice that for each step the fourths columns added to the fifths columns exactly equals one octave.

Table of first 43 Note Names, Hertz and Cents for cumulative fifths and fourths (A=110 Hz.)

This table shows the result of cumulative fourth and cumulative fifth intervals. Large (L) and small (s)intervals are shown from A at 110 Hertz. The cent values are in relation to A and the frequencies assume A=110 Hz.

Note name	Position	cents from A	L & s from A	Hertz	Steps	Note name	Position	cents from A	L & s from A	Hertz
A	I	-------	-----	110.000	00	A	I	-------	-----	110.000
E	V	0695.493	**3L+s**	164.385	01	D	IV	0504.507	**2L+s**	147.215
B	II	0190.986	L	122.830	02	G	bVII	1009.014	4L+s	197.021
F#	VI	0886.479	4L+s	183.558	03	C	bIII	0313.521	L+s	131.838
C#	III	0381.972	2L	137.156	04	F	bVI	0818.028	3L+2s	176.442
G#	VII	1077.465	5L+s	204.967	05	Bb	bII	0122.535	s	118.068
D#	#IV	0572.958	3L	153.153	06	Eb	bV	0627.042	2L+2s	158.013
A#	#I	0068.451	L-s	114.436	07	Ab	bVIII	1131.549	4L+3s	211.471
E#	#V	0763.944	4L	171.015	08	Db	bIV	0436.056	L+2s	141.508
B#	#II	0259.438	2L-s	127.784	09	Gb	bbVII	0940.563	3L+3s	189.383
Fx	#VI	0954.931	5L	190.961	10	Cb	bbIII	0245.070	2s	126.727
Cx	#III	0450.424	3L-s	142.687	11	Fb	bbVI	0749.577	2L+3s	169.602
Gx	#VII	1145.917	6L	213.234	12	Bbb	bbII	0054.084	2s-L	113.491
Dx	xIV	0641.410	4L-s	159.329	13	Ebb	bbV	0558.591	L+3s	151.887
Ax	xI	0136.903	2L-2s	119.052	14	Abb	bbVIII	1063.098	3L+4s	203.273
Ex	xV	0832.396	5L-s	177.912	15	Dbb	bbIV	0367.605	3s	136.022
Bx	xII	0327.889	3L-2s	132.937	16	Gbb	3bVII	0872.112	2L+4s	182.041
F3#	xVI	1023.382	6L-s	198.663	17	Cbb	3bIII	0176.619	3s-L	121.815
C3#	xIII	0518.875	4L-2s	148.442	18	Fbb	3bVI	0681.126	L+4s	163.027
G3#	xVII	0014.368	2L-3s	110.917	19	B3b	3bII	1185.633	3L+5s	218.182
D3#	3#IV	0709.861	5L-2s	165.755	20	E3b	3bV	0490.140	4s	145.999
A3#	3#I	0205.354	3L-3s	123.853	21	A3b	3bVIII	0994.647	2L+5s	195.393

This progression continues infinitely. Pi is both an irrational and a transcendental number. Each further step will produce two new and unique values and hence more intervals and positions. Notice that the patterns of note names and note positions repeat themselves every seven steps adding an extra # through the fifths and b through the fourths.

For each step the cents, intervals, and note positions from the fifths added to the corresponding cents, intervals and note names from the fourths result in an exact octave.

Eg. after nine steps 763.94 + 436.06 = 1200.00 cents; #V + bIV = VIII and 4L + (L+2s) = 5L +2s.

|Graph of this pattern and comparison to integer frequency ratios| |Chapter One| |Chapter Three|

You may have noticed that the intervals which are closest on the spiral of fourths and fifths tend to sound more consonant.

To download more technical information go to LucyTuning homepage extract from CHAPTER THREE of

Pitch, Pi, And Other Musical Paradoxes

SCALES ON THE LUCY SCALE

12tET

Equal Temperament twelve notes per octave (12tET) assumes that there is only one interval on which all musical scales are based. This interval is the semitone (100 cents). Twelve equal semitones make one octave. Two semitones make one whole tone.

LucyTuning

The naturals arranged in order of fifths gives: **F-C-G-D-A-E-B** *1 2 3 4 5 6 7* This is a chain of seven notes, an expanse of six steps. Hence the first digit in the code is 6/ None of these notes are missing, so the second digit is 6/0. Each scale begins on the notes in positions 1 through 7 as follows:

So for Scales Using All The Naturals.

Notice that each octave consists of 5 Large (L) and 2 small (s) intervals in the circular sequence LLsLLLs with each scale starting from a different position in the sequence.

1	2	3	4	5	6	7	8	Scale Coding	Greek Name & Interval Pattern	Indian Name
C	D	E	F	G	A	B	C	6/0/2	Ionian LLsLLLs	Bilaval
D	E	F	G	A	B	C	D	6/0/4	Dorian LsLLLsL	Kafi
E	F	G	A	B	C	D	E	6/06	Phrygian sLLLsLL	Bhairavi
F	G	A	B	C	D	E	F	6/0/1	Lydian LLLsLLs	Kalyan
G	A	B	C	D	E	F	G	6/0/3	Mixolydian LLsLLsL	Khamaj
A	B	C	D	E	F	G	A	6/0/5	Aeolian LsLLsLL	Asavari
B	C	D	E	F	G	A	B	6/0/7	Locrian sLLsLLL	-

Table of more than 60 scales For more information on LucyTuned ScaleCoding

conventional keyboard layout for modulation of pattern (.gif)

***Pitch Pi,....*|Chapter One| |Chapter Two|**

|LucyTuning homepage|

Some of the scales and intervals available using LucyTuning

Clicking in the rectangle area for each scale will trigger a LucyTuned **midi** file for you to hear

12tET.mid

More than sixty of the thousands of scales you can play using LucyTuning ® & LucyTuning technology

.mid 12tET

No.	Scale Names / Mode Code / Intervals	1	2	3	4	5	6	7
1	Ionian / *Bilaval* 6/0/2 (White notes on piano ascending from C up to B)	C L	D L	E s	F L	G L	A L	B s
2	Dorian (Western Name) 6/0/4 *Kafi (Indian Name) (Naturals D-C)*	C L	D s	Eb L	F L	G L	A s	Bb L
3	Phrygian / *Bhairavi* 6/0/6(Naturals E-D)	C s	Db L	Eb L	F L	G s	Ab L	Bb L
4	Lydian / *Kalyan* 6/0/1 (Naturals F-E)	C L	D L	E L	F# s	G L	A L	B s
5	Mixolydian / *Khamaj* 6/0/3 (Naturals G-F)	C L	D L	E s	F L	G L	A s	Bb L
6	Aeolian / *Asavari* 6/0/5 (Naturals A-G)	C L	D s	Eb L	F L	G s	Ab L	Bb L
7	Locrian 6/0/7 (Naturals B-A)	C s	Db L	Eb L	F s	Gb L	Ab L	Bb L
8	Melodic Minor 8/28/4	C L	D s	Eb L	F L	G L	A L	B s
8d	*(descends as Aeolian)*					s	Ab L	Bb L
9	Neapolitan 10/24810/6	C s	Db L	Eb L	F L	G L	A L	B s
10	Neapolitan Minor 10/48910/6	C s	Db L	Eb L	F L	G s	Ab (2L-s)	B s
11	Harmonic Minor 9/389/5	C L	D s	Eb L	F L	G s	Ab (2L-s)	B s
12	Hungarian Minor 10/3489/5	C L	D s	Eb (2L-s)	F# s	G s	Ab (2L-s)	B s
13	Romanian 9/389/4	C L	D s	Eb (2L-s)	F# s	G L	A s	Bb L
14	Hungarian Folk or Byzantine 10/3489/6	C s	Db (2L-s)	E s	F L	G s	Ab (2L-s)	B s
	Indian Name scales							
15	Bhairav 9/238/6	C s	Db (2L-s)	E s	F L	G L	A s	Bb L
16	Marva / Marvi 11/23458/6	C s	Db (2L-s)	E L	F# s	G L	A L	B s
17	Purvi bVI 11/34589/6	C s	Db (2L-s)	E L	F# s	G s	Ab (2L-s)	B s
18	Purvi bVII 11/235811/6	C s	Db (2L-s)	E L	F# s	G L	A s	Bb L
19	Todi bVI 11/458910/6	C s	Db L	Eb (2L-s)	F# s	G s	Ab (2L-s)	B s
20	Todi bVII 11/2581011/6	C s	Db L	Eb (2L-s)	F# s	G L	A s	Bb L
21	Persian 11/458910/7	C s	Db (2L-s)	E s	F s	Gb L	Ab (2L-s)	B s
22	Spanish Folk & Jewish Major 9/389/6	C s	Db (2L-s)	E s	F L	G s	Ab L	Bb L
23	Enigmatic / Verdi 15/2345789131 5/6	C s	Db (2L-s)	E L	F# L	G# L	A# s	B s
24	Stravinski example 10/24810/5	C L	D L	E L	F# s	G s	Ab L	Bb L
25	Whole Tone alternate 12/2468101 2/1	C L	D L	E L	F# L	G# L	A# L	B# (2s-L)
26	Hindi bVI & bVII 8/28/5	C L	D L	E s	F L	G s	Ab L	Bb L
27	Hindi #IV & #V 8/28/1	C L	D L	E L	F# L	G# s	A L	B s
28	Hindi #IV & bVII or Dominant 7th Lydian 8/28/3	C L	D L	E L	F# s	G L	A s	Bb L
29	Hindi 3 flats & bV 8/28/7	C L	D s	Eb L	F s	Gb L	Ab L	Bb L
30	Hindi 5 flats & bIV 8/28/9	C s	Db L	Eb s	Fb L	Gb L	Ab L	Bb L
31	Hindi bII, bIII & bVII 8/28/6	C s	Db L	Eb L	F L	G L	A s	Bb L
32	Damien Emanuel 10/48910/5	C L	D s	Eb (2L-s)	F# s	G s	Ab L	Bb L
33	Pseudo Turkish [illegible]	C [illegible]	Db [illegible]	Eb [illegible]	F [illegible]	Gb [illegible]	A [illegible]	Bb [illegible]

SOME OF THE PENTATONIC POSSIBILITIES

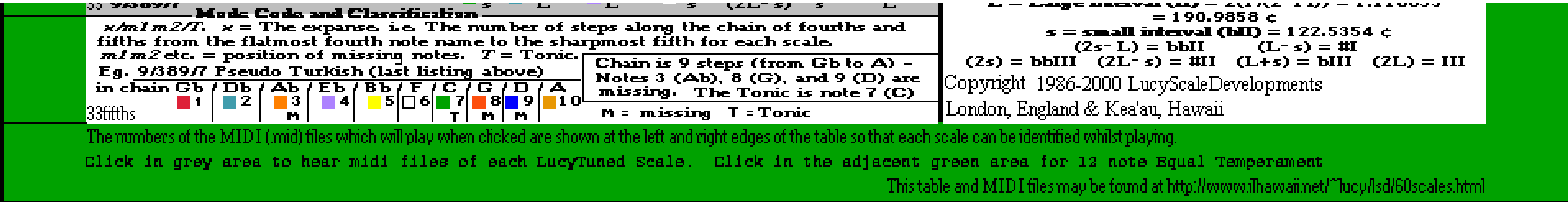
Blues Scales	1	2	3	4	5	No.
Blues bV 6/23/7	C (L+s)	Eb L	F s	Gb (2L)	Bb L	34
Blues #IV 9/25678/4	C (L+s)	Eb L	F (L-s)	F# (2L+s)	B s	35
Minor Blues 4/0/4	C (L+s)	Eb L	F L	G (L+s)	Bb L	36
Country and Old Chinese 4/0/1	C L	D L	E (L+s)	G L	A (L+s)	37
Scriabin 9/23458/6	C s	Db (2L-s)	E (L+s)	G L	A (L+s)	38
Oriental 5/4/3	C L	D (L+s)	F (2L)	A (L+s)	Bb L	39
Ancient Egyptian & Indian 6/56/3	C (2L)	E s	F L	G (L+s)	Bb L	40
Example of Slendro 11/4thru10/12	C (2s)	Ebb (2L-s)	F (2s)	Abb L	Bbb (2L-s)	41
Japanese Scales (JAPANESE)						
Ritusen 4/0/2	C L	D (L+s)	F L	G L	A (L+s)	42
Hirajoshi 6/35/4	C L	D s	Eb (2L)	G s	Ab (2L)	43
Kumoi 6/23/4	C L	D s	Eb (2L)	G L	A (L+s)	44
Iwato 6/34/7	C s	Db (2L)	F s	Gb (2L)	Bb L	45
Soft Ascend 6/23/6	C s	Db (2L)	F L	G (L+s)	Bb L	46a
Soft Descend 6/34/6	C s	Db (2L)	F L	G s	Ab (2L)	46d
Hard Ascend 4/0/4	C (L+s)	Eb L	F L	G (L+s)	Bb L	47a
Hard Descend 4/0/2	C L	D (L+s)	F L	G L	A (L+s)	47d

Example of Pelog 22/2346781112 1316thru21/9	C	Cx	Fb	F#	G	Ab	B	No.
	(2L-2s)	(4s-L)	(2L-2s)	s	s	(2L-s)	s	48

Some other 6&8 note scales

1	2	3	4	5	6	7	8	Mode Code & Classification	No.
C (L+s)	Eb L	F s	Gb (L-s)	G (L+s)	Bb L			7/23/7	49
C (L+s)	Eb L	F (L-s)	F# s	G (L+s)	Bb L			9/678/4	50
C L	D L	E L	F# L	G# L	A# (2s)			10/246810/1	51
C L	D L	E (2s)	Gb L	Ab L	Bb L			10/246810/7	52
C L	D L	E L	F# (L+s)	A s	Bb L			8/248/3	53
C s	Db (2L-s)	E L	F# (L+s)	A s	Bb L			11/2357811/6	54
C L	D s	Eb L	F (L-s)	F# s	G s	Ab L	Bb L	10/8910/5	55
C s	Db L	Eb L	F (L-s)	F# L	G# s	A L	B s	13/24781013/6	56
C s	Db L	Eb (L-s)	E s	F s	Gb L	Ab L	Bb L	10/8910/7	57
C L	D s	Eb L	F L	G s	Ab L	Bb (L-s)	B s	9/89/5	58
C (L-s)	C# s	D L	E L	F# s	G L	A L	B s	7/0/1	59
C L	D L	E s	F (L-s)	F# s	G L	A L	B s	7/0/2	60
C L	D L	E s	F L	G L	A s	Bb (L-s)	B s	7/0/3	61
C L	D s	Eb (L-s)	E s	F L	G L	A s	Bb L	7/0/4	62

Interval to note above is listed under each note name
L = Large interval (II) = 2^(1/(2*Pi)) = 1.116633



This diagram is also in the LucyTuned Lullabies (from around the world) booklet.

|Scalemaking| |More diagrams about LucyTuning| |LucyTuning homepage|

LucyScaleDevelopments presents extracts from:

"Pitch, Pi, and Other Musical Paradoxes (a practical guide to natural microtonality)"

by Charles E. H. Lucy

Recipe for homemade (or professionally manufactured) 3-dimensional Physical Modelling spiral for all meantone and similar tunings.

Ingredients:

Length of flexible wire (8 to 10 metres) (25 to 30 feet) or 5-8 wire coat hangers.

50 (Elastic) rubber bands.

31 wooden clothes pegs (clothes pins) or 31 tags or sticky labels for pegs.

5 one metre (3 foot) lengths of different colored string or wool..

One cylindrical can.

(For optional coloured note positions) Palette of paints and brushes. Pitch to colour

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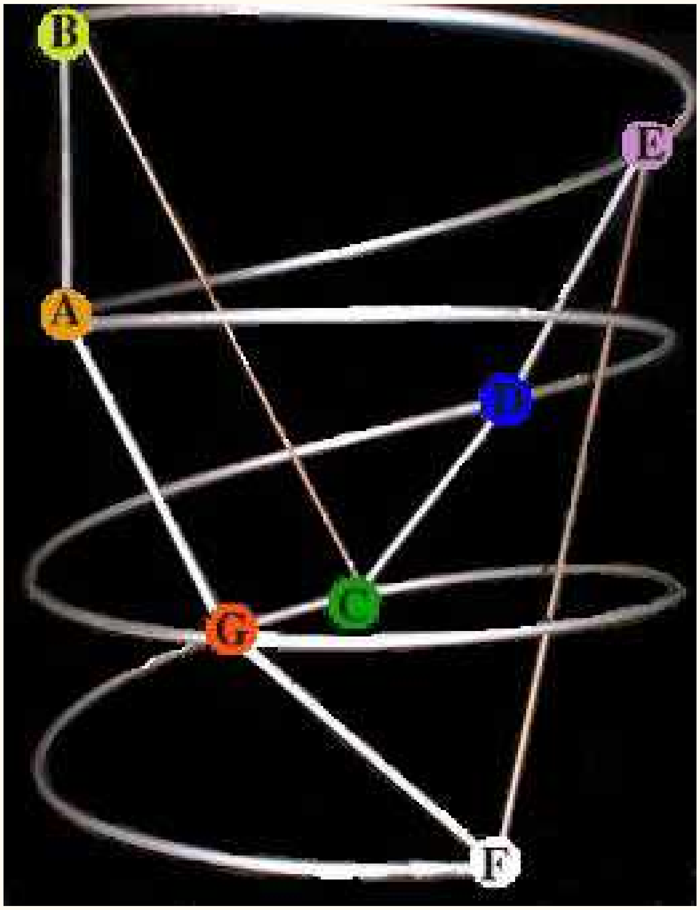
The professional version (a partial view)

Just All The Naturals

C-*L*-D-*L*-E-*s*-F-*L*-G-*L*-A-*L*-B-*s*-C



Hamster Baskets in England produce and sell this spiral for educational, demonstration, and experimental purposes.
Their version is made from steel and comes with copper *Large* and iron *small* fixable clip-on "intervals sticks", to join and stabilise the spiral and scales. Loops are welded onto the spiral, to which you can also attach colours, labels, and/or strings.
Although they are delivered adjusted for LucyTuning, by changing the radius of the spiral, any meantone or similar tuning and its scales can be represented (including Pythagorean).

Tools required:

One protractor.

Directions (for homemade version):

To prepare coat hangers:

Unbend coat hangers. Join ends of straightened wires end to end with tape or weld to provide a 8 to 10 metres continuous length of wire.

Construction from wire: Bend wire around a conveniently sized cylindrical tin (can) to form spiral spring shape of 8-12 inch (200-400mm) diameter.

Position, place or tie 31 elastic bands for markers at intervals of 208.65 degrees around the spiral spring using protractor.

Label thirty-one clothes pegs (pins). One for each note name: Fbb Cbb Gbb Dbb Abb Ebb Bbb Fb Cb Gb Db Ab Eb Bb F C G D A E B F# C# G# D# A# E# B# F## C## G##.

Attach labeled clothes pegs: Suspend the wire spiral with its elastic band markers by a loop bent from the wire after your last mark. Eg. From a lamp or ceiling. Attach the clothes pegs in the above sequence (at the marks, beginning at the end you started your angle measurements from. (the bottom of the spiral), and ascending by one fifth (208.65 degrees) each step.

There are many possible modifications for this basic design, eg. paint markings, color code pegs. blue tack, or chewing gum instead of elastic bands, although the angle and spiral shape should be precise.

Display various scales

1. *Greek Modes and Indian equivalents using only naturals,* attach a length of string to join the pegs: C-D-E-F-G-A-B and back to C.

You will notice that the Large intervals move upwards and one radian around the spiral. The small intervals move downwards and 68.45 degrees around the spiral.

Starting at each of the points and listing them in ascending order will give you the seven Greek modes (Indian names in brackets):

starting from C=Ionian (Indian Bilaval Scale); D=Dorian (Kafi); E=Phrygian (Bhairavi); F=Lydian (Kalyan); G=Mixolydian (Khamaj); A=Aeolian (Asavari; and B=Locrian Mode.

2. *Sharps and flats* Attach second and third strings of different colours in the following sequences:

1) G#-A#_B#-C#-D#-E#-F##-G#

2) Ab-Bb-C-D-Eb-F-G-Ab

You will see that you have generated the same shapes, but higher and lower on the spiral and with a small phase shift between them of (2s-L)=54.084" or 16.2252 degrees

(1) Up into the sharp tonalities [G# Major scale (Ionian) i.e. 8 sharps]

(2) Down into the flat tonalities [Ab Major (Ionian) Scale i.e. 4 flats]

3. *Hindi Scale* Tie strings to connect the following sequence: C-D-E-F-G-Ab-Bb-C

This produces the shape of one of the fundamental Hindi scales. You will notice that it follows the sequence of the Ionian mode up to G but diverges after that as though it was a flat Western scale having three flats. You could consider it as an Ab Major with the Eb sharpened to E in Western terms or as the root of five further modes following the pattern of four adjacent Large intervals and a single Large separated by two separate small intervals. (LLsLsLL).

4. *The Mysterious Middle Eastern Interval.* East of a line North/South from about where the Iron Curtain used to stand, there is a cultural area which uses an interval absent from Western music. This interval is called the sharp second and is the difference between two Large and one small interval ***(2L-s)***. Examples of this interval are found in many Indian, Hungarian, and Romanian scales. You can represent it on your model by joining Ab and B; Db and E; Eb and F#, or any other way which gives the same angles or phase.

I trust these instructions have given you a taste for the geometry of musical harmonics. This is just a beginning. You can build your own new musical scales by connecting the pegs or interval sticks in any way which makes sense to you, and produce new modes, scales, and harmonies. Have fun with it.

LucyTuning home